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MEASUREMENT OF AERODYNAMIC DERIVATIVES IN WIND TUNNELS AND IN FLIGHT

Part One

RESEARCH IN WIND TUNNELS

by
M. Scherer

Part Two

DETERMINATION IN FLIGHT OF THE AERODYNAMIC COEFFICIENTS OF AN AIRCRAFT BY THE STUDY OF ITS FREQUENCY RESPONSE

by
P. Mathé

December 1967

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TABLE OF CONTENTS

LIST OF FIGURES

PART ONE

LIST OF FIGURES (Concluded)

PART TWO

MEASUREMENT OF AERODYNAMIC DERIVATIVES
IN WIND TUNNELS AND IN FLIGHT

The first part of this report, which concerns wind tunnel research on aerodynamic derivatives, amplifies previous overall studies by examining critically the methods and facilities generally used at the present time for such work. Some progress is reported and some results thus made possible are presented. The second part is devoted to flight tests undertaken by a French firm after development of prototype apparatus.

INTRODUCTION

Our purpose is to give an insight into the current state of techniques for measuring aerodynamic derivatives in wind tunnels and in flight and to justify the extent to which they may be relied on in the domain of sonic and supersonic speeds by comparing the few available results.

The first part will deal with experimental methods in wind tunnels of moderate dimensions. With the help of the theory, these should furnish, at the preliminary stage, the aerodynamic data necessary for the discussion of stability and maneuvering.

However, in spite of the progress recently made in the techniques of calculating or measuring in wind tunnels, numerous problems remain, especially in the transonic domain.

The builder is thus led to undertake, a posteriori on the prototype machine, the measurement in flight of the aerodynamic coefficients, with the object of confirming or even completing the results which he was able to obtain in the course of studying the project.

The second part of this paper will be devoted to this second aspect of the problem.

PART ONE

RESEARCH IN WIND TUNNELS

M. Scherer

1.1 GENERAL

In a lecture given in 1954, Maurice Roy showed the possibilities of applying the classic theories of stability to experimental research in flight mechanics.[1] Several suggestions are expressed in it. They have served as a basis for research undertaken at the Office Nationale d'Etudes et de Recherches Aéronautiques (National Office of Aeronautical Study and Research).

In an AGARDograph[2] which appeared in 1955, Lee Arnold describes the principal installations established by the aeronautical research groups of the nations belonging to AGARD and gives their principles of operation.

In a more recent publication, written in 1959 for a work session of the AGARD Wind Tunnel Committee at Marseilles,[3] Orlick-Rückemann completely classifies the different methods. In addition, his paper contains 20 pages of bibliographic references, most of which are later than 1950. This demonstrated clearly the extent of the work to which this research has given rise in recent years.

However, by relying on these comprehensive studies, a rapid survey will suffice to bring us up to date on the state of the art in this field.

Generally speaking, it has been possible to overcome the primary difficulties encountered when setting up installations based on the different methods, thanks largely to the progress made in electronic techniques.

These installations are currently in everyday use in sub- and supersonic wind tunnels and efforts are now being directed toward making them more profitable. Hypersonic measurements have only recently been attempted. They will benefit from the experience already acquired.

In that which follows, the experimental techniques, classified by test method, will be examined in the usual order: uniform rotation, free oscillations, forced oscillations, and continuous oscillations.

1.2 UNIFORM ROTATION

This is the oldest procedure and also the most convenient for measuring the derivatives with respect to the angular velocity of the axis parallel to the wind. It can be considered from two points of view: forced rotation or free rotation.

1.2.1 In the case of forced rotation, the direct derivative of damping in roll, expressed by the coefficient C_{ℓ_p}, is determined by the slope of the curve of the rolling moment, plotted as a function of the rotation velocity.

This curve is most generally a straight line, which is an indispenable condition for obtaining accurate measurements with this method.

Measurements of moment, made in wind tunnels by keeping the flow velocity and the rotation velocity of the model constant, are by nature stationary. They are read from the standard measuring instruments of the wind tunnel. It is therefore possible, in many wind tunnels, to record them on printouts and punched tape, with a view to an automatic reduction of the data.

It is of interest to recall briefly the principles of arranging the balances (Fig. 1):

(1) The measurement of the fulcrum reaction was adopted at the O.N.E.R.A. by M. Bismut[4] for the first balance set up in 1950. The model is driven by an electric motor, to whose bedplate, mounted on two bearings, a dynamometer with resistance gauges is fixed.

(2) The direct measurement of the moment on the rotating shaft was adopted at two NASA installations, once by Brown and Heinke[5] and again by Wiggins.[6]

An assembly on the scale of the first installation has recently been set up at the O.N.E.R.A., employing a sting of normalized dimensions. This assembly is adapted to a jet size of $0.3 \times 0.3\ m^2$. The dynamometer has resistance gauges; the feed and measuring current pass through sliding contacts.

With the first arrangement, it is necessary to be sure that the parasitic moment due to friction on the bearings of the balance bed represents only a negligible fraction of the moment sought. With the second arrangement, it is necessary to verify that the sliding contacts do not interfere with the measuring and that the axis of measurment coincides with the rotation axis.

Experience shows that the results furnished by the two devices are equivalent.

These balances are perfectly suited to measurements on models of missiles in transonic and supersonic wind tunnels of small dimensions.

The moment to be measured is very small, and it is indispensable that complete stability of the rotation velocity be assured, at the risk of introducing into the measurements inertial stresses due to angular acceleration.

The very simple apparatus described below, devised by M. d'Humieres at the O.N.E.R.A., gives the velocity easily and with great precision and simultaneously permits the visible, permanent control of its stability. The signal from a calibrated generator is immobilized on the screen of a dual-beam oscilloscope, whose second channel is launched by a lancehead connected to the shaft. The velocity is stabilized and adjusted to the fixed rate when the two signals appear immobile on the screen. The least deviation in adjustment causes the displacement of one image with respect to the other (Fig. 2).

1.2.2 The measurement of the cruciform derivatives of the yawing moment C_{n_p} and of the drift force C_{y_p} can also be determined from uniform rotation tests. The use of two revolving dynamometers attached to the shaft seems the most advisable here (Fig. 3). The quantities to be measured are very small. In the case of ordinary airplane models, they rarely exceed one tenth of the rolling moment. Also, the dissymmetries in the shapes of the models, whose relative importance increases for small-scale models, lead to parasitic stresses which are of the order of the quantities to be measured. However, since the value of these disturbances is independent of the direction of rotation, it is possible to eliminate the majority of them by turning the model first in one direction, then in the other. Nevertheless, a slight correction is still necessary to allow for the residual values arising from the deformation of the sting.

The data for this correction are furnished by:

(a) Measurements of the deformation of the sting under static loads;

(b) The derivative of position C_{n_j} of the yawing moment with respect to the sideslip angle;

(c) A test without wind, with the balance rotating uniformly. This will give the amount of unbalance;

(d) The values of moment and force obtained during tests with wind, which allow the deformation to be calculated.

A supplementary correction may arise from a possible lack of perpendicularity between the axis of measurement and the rotation axis. Its importance is due to the differences in order of magnitude, already indicated, between the rolling and yawing moments. This correction can be determined from a static calibration.

Unfortunately, the method of forced uniform rotation, which is easy to put into practice, is practical only if the angle of incidence is close to zero.

It seems difficult to use in burst-type wind tunnels.

1.2.3 Free Rotation

The second aspect of the method will be mentioned here only as a reminder. The measurement of the rotation velocity of a model made dissymmetrical furnishes a relation which links the effect of the dissymmetry to the damping moment caused by the rotation.

A simple method of measuring this velocity consists of taking a center section or empennage and using this signal in the same way as described in Section 1.2.1.

1.3 FREE OSCILLATIONS

This method, which is as old as the preceding one, has been practically abandoned in the incompressible domain. On the other hand, it finds numerous applications in the transonic and supersonic domains, mainly for the purpose of determining the coefficient of aerodynamic influence, which occur in the buffeting of the structures. A report on this question by R. Dat, of the O.N.E.R.A., was published in 1958 (Fig. 4).[7]

The models, often of the wall type, are mounted on a suspension consisting only of elastic links with one degree of freedom, which are firmly embedded. The moduli of rigidity of these links are high. They allow for significantly reduced frequencies with low structural damping.

The frequencies are measured with great precision by pulse counters, making possible the determination of aerodynamic responses, which are generally small in relation to the rigidity of the suspension.

The decrement is usually measured by the graphic recordings of the oscillations, often obtained after filtering from the original recording on magnetic tape.

But there are also automatic devices, such as the "dampometer" of Olson and Orlick-Rückemann, which is already old (1954), and the apparatus created recently by Bratt and Wight of the N.P.L., which consists of electronic gates and an analog integrator. The oscillation to be analyzed is recorded on magnetic tape. The device permits the damping to be measured over a predetermined number of oscillations in any given zone of the band recorded.

1.3.1 Among the free-oscillation installations recently put into service, examples can be cited which are on widely differing scales:

(1) In the 0.75-m × 0.4-m (30-in. × 16-in.) burst-type wind tunnel of the N.A.E., the damping-in-roll balance of L. T. Conlin and K. J. Orlick-Rückemann.[8]

(2) In the transonic Great Wind Tunnel (8 m in diameter) of the O.N.E.R.A. at the Modane-Avrieux, there is an assembly (Fig. 5) for measuring the damping around the three axes, in which each degree of fredom around each axis is librated successively. The weight of the models tested can be as high as one ton; however, on this scale it is very difficult to completely eliminate the play and friction of the suspension. M. Vaucheret, in an unpublished theoretical study, has shown that it is easily possible to correct their effect, provided that the amplitudes are at least five times greater than the play. If this condition is fulfilled:

 (a) The slope of the envelope of the maxima of elongation is the same as that of the linear system without play;

 (b) The inverse of the $\frac{1}{2}$ period, taken between two zero values of the elongation, and the corresponding relative play are connected by a linear relation.

(3) The first assemblies for measuring damping in pitch on models of reentry bodies at the A.E.D.C. by C. J. Welsch, R. L. Ledford, L. K. Ward, and J. P. Rhudy.[9]

1.4 CONTINUOUS OSCILLATIONS

This method is generally associated with the preceding one and their areas of application are the same.

It is advantageous to continue the movement at the frequency of the free oscillations, then to eliminate the exciting force, for the effects of the transient terms, which are due to the response time of the measuring instruments, no longer appear on the recording.

The measurements obtained from continuous oscillations at one degree of freedom, namely the amplitude and phase of the motion, when compared with those of the exciting force, permit the calculation of the aerodynamic derivatives related to this motion. If the other degrees of freedom of the model are locked by low-force dynamometers, their response will furnish the corresponding cruciform derivatives.

This method is applicable to large-scale models equipped with control surfaces.

The measuring devices are the same as those used in the forced oscillation method, which will be examined in the following section.

1.5 FORCED OSCILLATIONS

At the present time, numerous low-velocity wind tunnels have measuring installations based on this method. Some of them are described in the AGARDograph by Lee Arnold already cited.

We shall not discuss these installations, which are already old and well known.

At this session of the AGARD, Harleth G. Wiley and Albert L. Braslow have discussed the new installations for transonic and supersonic testing at Langley Field.

We shall therefore limit ourselves here to making a few observations on the installations recently put into service at the O.N.E.R.A., in the transonic and supersonic wind tunnels of Chalais-Meudon and La Courneuve.

These installations have already been the subject of two papers, one presented to the AGARD in 1959 at Marseilles and the other to the 4th European Aeronautical Congress at Cologne in 1960.

The desired goal was a measuring assembly which would meet the following conditions:

(a) Measure all the aerodynamic derivatives occurring in the stability calculations;

(b) Be adapted to wind tunnels of moderate dimensions (30 cm × 30 cm);

(c) Be easy to set up;

(d) Not require troublesome operations for the reduction of wind tunnel data.

The oscillation method seemed most apt to fulfill these conditions, for various reasons:

The technique of dynamometers with resistance gauges, developed and perfected at the O.N.E.R.A. for several years, led to the creation of normalized sting balances, which give the six components of aerodynamic stresses with a high degree of precision.

It was relatively easy to arrange these stings on supports oscillating at one degree of freedom around the axes of pitch, yaw, and roll, successively.

The measuring instruments used for stationary tests were suitable for dynamic measurements, provided that the gauges were supplied with alternating current of a frequency equal to that of the oscillation.

This procedure had already been applied with success by M. Bismut to the transversal stability balance of the low-velocity wind tunnel of the O.N.E.R.A. at Cannes.[1]

The available means of automatic calculation could enable all the numerical operations necessary for the reduction of wind tunnel data to be carried out rapidly and with great accuracy.

The first balances put into service give the aerodynamic derivatives of pitch and yaw. Their design is shown in Figure 6.

The same assembly serves for both longitudinal and transversal measurements. To go from one to the other, it is necessary only to turn the model 90 deg about the axis of the sting.

The location of the rotation axis aft of the model results in a support of small dimensions in the jet. It therefore causes only negligible disturbances to the model in the subsonic domain, and in supersonic flow it does not hinder in any way the inducement of the blast.

The position detector, whose function will be specified later, is equipped with a measuring device with resistance gauges (Fig. 7).

The source of the sinusoidal of analyzing current is formed by a series of standard electronic devices. The schematic of the entire installation is given in Figure 8.

The tests consist of several series of measurements both with and without wind, whose respective objects are the determination of the components of the forces in phase and in quadrature with the motion, the amplitude of the motion, and the forces of inertia.

During these experiments, the voltage of the feed current of the gauges (or analyzing current), the frequency of the oscillations, and the velocity of the wind, where applicable, are kept constant at each measuring point. The phase of the current is regulated successively to coincide then to be in quadrature with the forced motion.

The vectorial representation of the adjustments and measurements is given in Figure 9.

The phase adjustments are performed manually by means of an electronic phase shifter and are controlled visually:

(a) In the case of coincidence, on an oscilloscope, by making appear on the screen a Lissajous figure formed by two signals, one emitted by the position detector and the other by the analyzing current.

(b) In the case of quadrature, by canceling the component of the angle of oscillation shown by the measuring instrument, since adjustment by the Lissajous figure is no longer sufficiently precise.

This last operation is the most delicate, for, except in certain special cases, its precision determines that of the measurements of the derivatives with respect to angular velocity, which are the essential objects of these tests.

However, experience shows that the zero position of the measuring instruments is not well known during the experiment. There results a phase error which, given the importance of the terms in phase with the forced motion, introduces (Fig. 10) an inadmissible error OH into the measurement of the term in quadrature ON = OK.

A method of symmetrical measurements allows this error to be eliminated. A first series of measurements is made, arbitrarily choosing a false zero 0_1 near the origin. Then the phase of the current is reversed and adjusted, while bringing the indicator of the measuring instrument back to the position of the original false zero 0_1. In both cases, the terms of error OH and OH' are equal, while the projection OL of the quantity to be measured changes sign. The half-difference of the readings then gives the desired quantity ON. The measurement is independent of the position of the false zero 0_1, a fact confirmed by experience.

The measuring of the amplitude of the motion requires a few observations.

Given the order of magnitude of the angle to be measured, the elasticity of the drive mechanism causes an augmentation of amplitude which is a function of the stresses undergone by the model and which can exceed 10 percent of the nominal value. It is therefore indispensable to provide for its measurement.

In addition, the inevitable compromise between the sensitivity of the dynamometer and its rigidity brings about a deformation of the sting which escapes the position detector. This causes amplitude increments in the incidence of the model and in the translation of its center of gravity (Fig. 15).

For the purpose of calculating them, two hypotheses have been formulated:

> The first, that the dynamic deformation can be represented by the effect of a localized static load;
>
> The second, that this dynamic deformation occurs in phase with the force applied.

These hypotheses have been confirmed by measurements made without wind, in which the model was represented by a weight with a simple geometric shape.

Figure 11 gives a vectorial representation applicable to either one of the two parameters of deformation. It shows that components of the forces of inertia due to the deformation of the sting are included in the measurements in quadrature.

Finally, the calculation of the desired aerodynamic derivatives, as related to the axis passing through the center of gravity of the aircraft, takes the form of linear relations with complex coefficients between the different parameters measured (see Supplement).

The rather troublesome calculations are performed very rapidly and with great accuracy by the machines at the Automatic Computation Center of the O.N.E.R.A.

The precision of the measurements is as follows:

(1) The reading threshold of the frequency is 0.2 percent and the precision about 0.5 percent, limited principally by the stability of the drive motor.

(2) Angles, forces, and moment are obtained by elongation readings. The precision of the method is at the present time limited by the reading devices used. It is $\frac{1}{100}$ in five ranges of sensitivity going from 1 to 0.01.

This precision will be improved when the measurements are made by a potentiometer method.

(3) The adjustment of the phase to the quadrature is obtained to within $\frac{1}{20}$ of a degree.

1.6 REMARKS

Under certain conditions, the position of the model with respect to the axis of measurement permits the mass and aerodynamic moments to be balanced between them. This balance can be total at a well-determined frequency whose value is a function of the various parameters of the test.

In this case, the measurement of the coefficient of damping in pitch $C_{m_{q_1}}$ is obtained with maximum precision, thus:

> An error of 8 deg in the phase adjustment introduces an error of only 1 percent into the measurement of the term in quadrature; the measurement of the stationary derivative C_{m_i}, which figures in the calculation of $C_{m_{q_1}}$, is obtained by a zero method; the deformations are less extensive.

The improvements brought about by this "zero method" are established by the comparison of the results obtained for forward centering, where this balance is achieved, and for aft centering, where it cannot be. The comparison will concentrate first on the measurement of the stationary coefficients $C_{z_{i_1}}$ and $C_{z_{q_1}}$, whose values are well known otherwise, then on the measurement of $C_{z_{q_1}}$ and $C_{m_{q_1}}$.

The results obtained at Mach 2 will be discussed as examples:

(a) An examination of them shows that in forward centering, with the conditions necessary for balance being satisfied, the error in the nonstationary measurement of the coefficient $C_{z_{i_1}}$ does not exceed

± 2.5 percent. As the frequency parameter ωR increases, this at first remains constant; then after a certain value of ωR, it begins to increase (Fig. 12). The results obtained with aft centering led, after a first analysis, to an average error of 7 percent for this same coefficient. The static coefficients of deformation were then measured again on a more rigidly standardized bedplate, and the difference was brought back to within the preceding margin of error.

(b) The precision of the measurement of the coefficient $C_{m_{\dot{i}_1}}$ is established by comparing the locations of the center in both cases of centering. In forward centering, the variation from the stationary value does not exceed 0.3 percent for the first four values of the frequency. It is more than 1 percent for aft centering, in spite of the improvement in the coefficients of deformation brought about by the second standardization.

In the present state of the technique ($\delta\phi$ on the order of $1/20$ of a degree), there is reason to expect an error of 6 percent of the central cord of the wing then determining the position of the axis corresponding to a zero value of $C_{z_{q_1}}$. Consequently, an error of the same order is to be expected in calculating the change of axis relative to $C_{m_{q_1}}$.

The values of $C_{m_{q_1}}$ measured at forward centering vary by less than 5 percent from the values found in the vicinity of the balancing frequency. The errors in measurement are amplified by the low value of the ratio $J(M_1)$ between the corrected aerodynamic moment and the gross value:

$$\overline{J(M_1)} = \frac{\frac{1}{2}\rho V^2 S \ell C_{m_{q_1}} (q_1 \ell / V)}{J(M_1)}$$

The value chosen for $C_{m_{q_1}}$ contains an average error of 3 percent, if it is assumed that the direct measurements are known to within 1 percent.

The experimental points found at aft centering show a dispersion of 15 percent. This results, on the one hand, from errors due to the phase adjustment, magnified by the value of the quotient $\frac{1}{2}\rho V^2 S \ell C_{m_{q_1}} (q_1 \ell / V)/R(M_1)$;

and on the other hand from reading errors, whose significance is cumulative since the values measured are small. The most accurate measurement in this case is the one made at the lowest frequency. It may still, however, contain an error of ±8 percent.

The calculation of $C_{m_{q_1}}$ at aft centering from the measurements at forward centering makes evident a difference of 8 percent between the two results, which do not cross-check except within their margins of error.

PART TWO

DETERMINATION IN FLIGHT OF THE AERODYNAMIC COEFFICIENTS OF AN AIRCRAFT BY THE STUDY OF ITS FREQUENCY RESPONSE

P. Mathé

2.1 GENERAL

The methods for determining the aerodynamic coefficients of an aircraft by tests in flight are numerous: stabilized sideslip flights, studies of longitudinal and transversal free oscillations (period, damping), study of the limiting velocities of roll, etc.[10–14]

All these relatively simple experimental methods have the disadvantage of furnishing only partial results, and often at the price of simplified hypotheses which are more and more difficult to justify with modern aircraft.

The study of the frequency response of an aircraft to a sinusoidal order of any one of its control surfaces is in large part not subject to these criticisms. In particular, it furnishes all the aerodynamic coefficients of an aircraft after a single series of tests, and therefore in a very homogeneous way.[15]

If this method is still relatively little used, it is largely because it requires a certain number of "precautions," experimental as well as in the interpretation and application of the gross results of the tests.

2.2 STUDY OF THE FREQUENCY RESPONSE — EXPLANATION OF THE METHOD

The motion of an aircraft about its center of gravity is governed by a system of differential equations which may always be assumed to be linear if the amplitudes are sufficiently low. Moreover, this linearity supposes the mutual independence of longitudinal and transversal motion, a fact which has been confirmed for lesser motions and greatly simplifies matters, since the two types of motion may thus be studied separately. We are going to describe the method in the most complicated case: that of the transversal motion.

The linearized equations of transversal motion are

$$mV\left[-\frac{dj}{dt} + r_1 - (i - \epsilon)\,p_1\right] = Y_{j_1} j + Y_{\delta_1} \delta + mg\phi_1 + \left(Y_{\alpha_1} \alpha_1\right) \qquad (1)$$

$$A \frac{dp_1}{dt} - L_{j_1} j - L_{p_1} p_1 - L_{r_1} r_1 - L_{\delta_1} \delta - \left(L_{\alpha_1} \alpha\right) = 0 \qquad (2)$$

$$C \frac{dr_1}{dt} - N_{j_1} j - N_{p_1} p_1 - N_{r_1} r_1 - N_{\delta_1} \delta - \left(N_{\alpha_1} \alpha\right) = 0 \qquad (3)$$

with the relation

$$\frac{d\phi_1}{dt} = p_1 + (i - \epsilon) r_1 \qquad (4)$$

To a sinusoidal order δ:

$$\delta = \Delta \cos \omega t$$

will correspond a resulting motion of the aircraft - after the transient state has been damped - characterized by velocities of roll and yaw and sideslips of the form:

$$p_1 = P \cos \left(\omega t + \phi_{p\delta}\right)$$

$$r_1 = R \cos \left(\omega t + \phi_{r\delta}\right)$$

$$j = J \cos \left(\omega t + \phi_{j\delta}\right)$$

$$\phi_1 = \phi \cos \left(\omega t + \phi_{\phi\delta}\right)$$

or, by setting

$$a_p = \frac{P}{\Delta} \cos \phi_{p\delta} \qquad b_p = -\frac{P}{\Delta} \sin \phi_{p\delta}$$

$$a_r = \frac{R}{\Delta} \cos \phi_{r\delta} \qquad \text{etc...}$$

$$p_1/\Delta = a_p \cos \omega t + b_p \sin \omega t$$

$$r_1/\Delta = a_r \cos \omega t + b_r \sin \omega t$$

$$j/\Delta = a_j \cos \omega t + b_j \sin \omega t$$

By making the terms which are factors of sin ωt and cos ωt equal to 0 in Equations (1), (2), (3), and (4), the equations in Y_{j_1}, Y_{δ_1}, L_{j_1}, L_{p_1}, etc. are obtained:

$$mV\left[-\omega b_j + a_r - (i - \epsilon)a_p\right] - Y_{j_1} a_j - mga_\phi - Y_{\delta_1} = 0 \tag{1a}$$

$$mV\left[\omega a_j + b_r - (i - \epsilon)b_p\right] - Y_{j_1} b_j - mgb_\phi = 0 \tag{1b}$$

$$A\omega b_p - L_{j_1} a_j - L_{p_1} a_p - L_{r_1} a_r - L_{\delta_1} = 0 \tag{2a}$$

$$-A\omega a_p - L_{j_1} b_j - L_{p_1} b_p - L_{r_1} b_r = 0 \tag{2b}$$

$$C\omega b_r - N_{j_1} a_j - N_{p_1} a_p - N_{r_1} a_r - N_{\delta_1} = 0 \tag{3a}$$

$$-C\omega a_r - N_{j_1} b_j - N_{p_1} b_p - N_{r_1} b_r = 0 \tag{3b}$$

with the relations:

$$b_\phi = \frac{1}{\omega}\left[a_p + (i - \epsilon)a_r\right] \tag{4a}$$

$$a_\phi = \frac{1}{\omega}\left[b_p + (i - \epsilon)b_r\right] \tag{4b}$$

After the elimination of a_ϕ and b_ϕ, we thus obtain a system of six linear equations with ten unknowns, which are the aerodynamic coefficients Y_{δ_1}, Y_{j_1}, L_{j_1}, L_{p_1}, L_{r_1}, L_{δ_1}, N_{j_1}, N_{p_1}, N_{r_1}, and N_{δ_1}. That is to say that theoretically, it is sufficient to know the quantities a and b, corresponding to two distinct values of ω in order to have more equations than unknowns and therefore in principal to be able to determine all the aerodynamic coefficients.

In actual fact, this method is unfeasible for two reasons: the first one is that, obviously, it would require a precision in the measurement of the quantities a and b which is absolutely impossible to obtain; the second is that sideslips are difficult to measure dynamically, and therefore the quantities a_j and b_j are not well known.

This last difficulty is avoided by not measuring the sideslips, but calculating them from the relations (1a) and (1b), in which Y_{j_1} and Y_{δ_1} are assumed to be known.

longitudinal motion; the normal acceleration γ_n in the case of the study of longitudinal motion.

The sideslip measurements, by anemometric as well as mechanical probes, turned out to be unusable because of the considerable lags occurring in these devices, which were not particularly adapted to this type of measurement. It is possible to remedy this disadvantage, either by calibrating the equipment in a wind tunnel (which may present some difficulties in the case of supersonic speeds) or, more simply, by using a device especially adapted to dynamic measurements (anemometric probe equipped with pressure pickups with a very short response time).

On the other hand, the velocities of roll, yaw, and pitch are very accurately measured by rate gyros, whose dynamic calibration can easily be established on the ground. This calibration permits the corrections to be made in the measurements of the various rotation velocities to be computed (Fig. 16).

These corrections are far from being negligible, but are known with a good degree of accuracy.

2.3.3 Carrying Out the Tests

One of the principal difficulties of the tests, at least for the pilot, is the stabilization of the velocity, above all in the transonic domain, where each one of the aerodynamic coefficients of the aircraft varies rapidly with the Mach number.

During each flight, it is possible to carry out a maximum of about fifteen tests at different frequencies, for it is indispensable to wait for the most perfect stabilization of the oscillations of the airplane.

2.3.4 Analysis of the Tests

It is necessary to make about ten measurements per test point at different frequencies, transversally as well as longitudinally.

The various parameters (p_1, q_1, r_1) are photographically recorded, which permits all the incorrect tests (turbulence, incomplete stabilization) to be eliminated by simply examining the recorded tapes.

Amplitudes and phase shifts can be measured directly from the photographic recording, but the direct analysis of yards of tapes after a series of tests does not make the method very attractive. The use of a machine which furnishes a digital conversion of the quantities read from the recorded tapes

These terms do not occur in general except as corrective terms. It is therefore sufficient to use the values furnished by wind tunnel tests.

We thus obtain for each frequency tested two systems of two equations with four unknowns:

(roll)

$$A\omega b_p - L_{j_1} a_j - L_{p_1} a_p - L_{r_1} a_r - L_{\delta_1} = 0$$

$$-A\omega a_p - L_{j_1} b_j - L_{p_1} b_p - L_{r_1} b_r = 0$$

(yaw)

$$C\omega b_r - N_{j_1} a_j - N_{p_1} a_p - N_{r_1} a_r - N_{\delta_1} = 0$$

$$-C\omega a_r - N_{j_1} b_j - N_{p_1} b_p - N_{r_1} b_r = 0$$

The problems posed by the dispersion of the measurements are eliminated by performing a rather large number N of tests at different frequencies. Two systems are then obtained - one relative to roll, the other to yaw - of 2N linear equations with four unknowns, which are solved by the standard method of least squares.

2.3 PERFORMANCE OF THE TESTS

2.3.1 Obtaining a Sinusoidal Displacement of the Control Surface

The airplanes with which we carried out these tests all had electrohydraulic servo-controls. We therefore had only to connect these servo-controls to the output of a sinusoidal voltage generator, whose amplitude and frequency adjustment - in small increments - was controlled by the pilot.

Let us note that, when experimenting with frequencies relatively distant from the natural frequency of the airplane, it is necessary to have a very "pure" displacement of the control surface if it is desired to obtain a response from the airplane which can be utilized directly, i.e., is exempt from harmonics of the signal.

2.3.2 Parameters Recorded

These are:

The displacements of the control surface; the velocities of yaw and roll for transversal motion; the velocity of pitch for the study of makes them directly available to standard electronic ordinators and permits a not inconsiderable gain in time and accuracy.

A much more elegant method would consist of making a magnetic recording of the various parameters during the flight and then making a harmonic analysis of it, using the standard methods of analog computation.

Further utilization of the results - the calculation of the aerodynamic coefficients by the method of least squares - is presently carried out by electronic computers.

2.4 DISCUSSION OF THE METHOD ACCURACY

It is very difficult to calculate this accuracy, which depends not only on that of the measurements, but also on the number and dispersion of the tests.

2.4.1 Accuracy of the Measurements

This obviously depends on the precision of the instruments used, but also on the order of magnitude of the variables measured. Here there is a compromise to be made. If the amplitudes of the variables are too low, their precise measurement is that much more difficult, since they are then easily distorted by atmospheric turbulence. If the amplitudes are too high, the hypothesis of the linearity of the equations of motion of the aircraft is no longer valid. Practically speaking, we chose the displacement amplitudes of the control surfaces so as to obtain velocities of yaw and pitch on the order of 3 deg to 5 deg per second and velocities of roll on the order of 15 deg to 25 deg per second. The amplitude of angular motion was limited to 10 deg in roll.

2.4.2 Number of Tests

The method does not require the use of more than about ten different frequencies. These frequencies are chosen uniformly within a relatively narrow band around the natural frequency F of the airplane.

In practice, only the frequency band lying between half of F and twice F is explored. Tests carried out at higher or lower frequencies are in effect of no use (very small gain, phase varies very little with frequency).

In conclusion:

Concerning measurements made in flight, the essential interest of the method lies in its "global" character, in that it does not furnish individually one or another of the aerodynamic coefficients of the aircraft, but rather the matrix of these coefficients. This result is very important, at least from the point of view of the aviator, who, in many cases, desires nothing more than a coherent sum of coefficients which will give him the best picture of the general behavior of his airplane and permit him to study certain special problems, such as inertial coupling, adaptation of automatic pilots, etc. In this respect, the individual accuracy of each one of the coefficients is less important than the coherence and global accuracy of the whole.

CONCLUSION

This rapid survey has shown how the various test procedures currently in use in the subsonic domain may be extended to transonic and supersonic wind tunnels of moderate dimensions.

(1) The experiments in forced rotation parallel to the wind, which are particularly interesting for their ease of execution, are perfectly adapted to measurements, not only on airplane models, but also on models of missiles, even those of small dimensions.

(2) The use of free-oscillation test setups is especially indicated for two types of applications:

(a) The audio frequency measurement of the coefficient of stability for large-scale models or even on the actual machine;

(b) Buffeting studies of structures. In this case, the models are generally small and the frequencies high. These measurements are often preceded by continuous oscillation tests.

(3) Forced oscillation balances are beginning to come into everyday use. They have the advantage of furnishing all the coefficients, in addition to which they are especially easy to set up in small wind tunnels.

The difficulties encountered in the adjustment of these balances have been overcome, thanks to lighter models, the use of standardized equipment for stationary measurements, and the creation of devices assuring very fine phase adjustments. There is still progress to be made, notably in shortening the length of the tests and in adapting the method to burst-type wind tunnels.

(4) Concerning the principles of measurement in flight, there is, to our knowledge, practically no new element to add to the methods already described in existing literature.

(5) The continuous oscillation method, discussed as an example in Part 2, is applicable to transonic flight. The procedure of excitation by the servo-control appears attractive for its flexibility. However, there is still progress to be made on the practical level to facilitate the analysis of the tests. It would seem that modern techniques of recording and telemetering should permit the standardization of the methods for

utilizing tests in flight and in the wind tunnel. The results of the measurement of aerodynamic derivatives obtained by one or the other of these methods cross-check within the limits of their own margin of error, which is on the order of 10 percent.

LITERATURE CITED

1. Maurice Roy, La Stabilité Transversale de Vol et Quelques Recherches de l'O.N.E.R.A. (The Transversal Stability of Flight and Some Investigations Made by the O.N.E.R.A.), Office Nationale d'Etudes et de Recherches Aéronautiques Publication No. 69, 1954.

2. Lee Arnold, Dynamic Measurements in Wind Tunnels, AGARDograph No. 11, 1955.

3. K. J. Orlick-Rückemann, Methods of Measurement of Aircraft Dynamic Stability Derivatives, N.A.E. Report LR 254, 1959.

4. M. Bismut, Mesure Statique des Dérivées Aérodynamiques de Roulis (Static Measurement of the Aerodynamic Derivatives of Roll), La Recherche Aéronautique, No. 30, 1952.

5. C. E. Brown and H. S. Heinke, Jr., Preliminary Wind Tunnel Tests of Triangular and Rectangular Wings in Steady Roll at Mach Numbers of 1.62 and 1.92, National Advisory Committee for Aeronautics TN 3740, 1959.

6. J. W. Wiggins, Wind Tunnel Investigation at High Subsonic Speeds to Determine the Rolling Derivatives of Two Wing Fuselage Combinations Having Triangular Wings, Including a Semi-Empirical Method of Estimating the Derivatives, National Advisory Committee for Aeronautics RM L 53L18a, 1954.

7. R. Dat, Maquettes et Equipment de Mesure pour la Détermination en Soufflerie des Forces Aérodynamiques Instationnaires (Models and Measuring Equipment for the Determination of Nonstationary Aerodynamic Forces in Wind Tunnels), AGARD Report 220, 1958.

8. L. T. Conlin and K. J. Orlick-Rückemann, Comparison of Some Experimental and Theoretical Data on Damping-in-Roll of a Delta-Wing Body Configuration at Supersonic Speeds, N.A.E. Report LR 266, 1959.

9. C. J. Welsh, et al., Dynamic Stability Tests in Hypersonic Tunnels at Large Model Amplitudes, AEDC TR 59-25, 1959.

10. W. O. Brenhaus and L. Segel, Méthodes de Réponse Dynamique, Manuel d'Essais en Vol de l'AGARD (Methods of Dynamic Response, in the AGARD Flight Test Manual), Vol. II, Chap. 10, 1956.

11. C. L. Muzzey and E. A. Kidd, Mesure et Interprétation des Résultats d'Essais en Vol pour l'Etude de la Stabilité et du Contrôle Dynamique (Measurement and Interpretation of the Results of Flight Tests for the Study of Stability and Dynamic Control), AGARD Flight Test Manual, Vol. II, Chap. 11, 1956.

12. G. Leblanc, Un Exemple de la Détermination des Principaux Coefficients Aérodynamiques à Partir des Essais en Vol (An Example of the Determination of the Principal Aerodynamic Coefficients Based on Flight Tests), AGARD Report 189, 1958.

13. J. K. Zbrozek, On the Extraction of Stability Derivatives from Full-Scale Flight Data, AGARD Report 190, 1958.

14. C. H. Wolowicz and E. C. Holleman, Stability Derivative Determination from Flight Data, AGARD Report 224, 1958.

15. M. Scherer, et al., Détermination en Vol des Dérivées Aérodynamiques et de l'Efficacité des Gouvernes d'un Avion à Partir de ses Réponses a des Sollicitations Harmoniques Entretenues par le Pilote (Determination in Flight of the Aerodynamic Derivatives and of the Efficiency of the Control Surfaces of an Aircraft, Based on Its Responses to Harmonic Solicitations Maintained by the Pilot), La Recherche Aéronautique, No. 56, 1957.

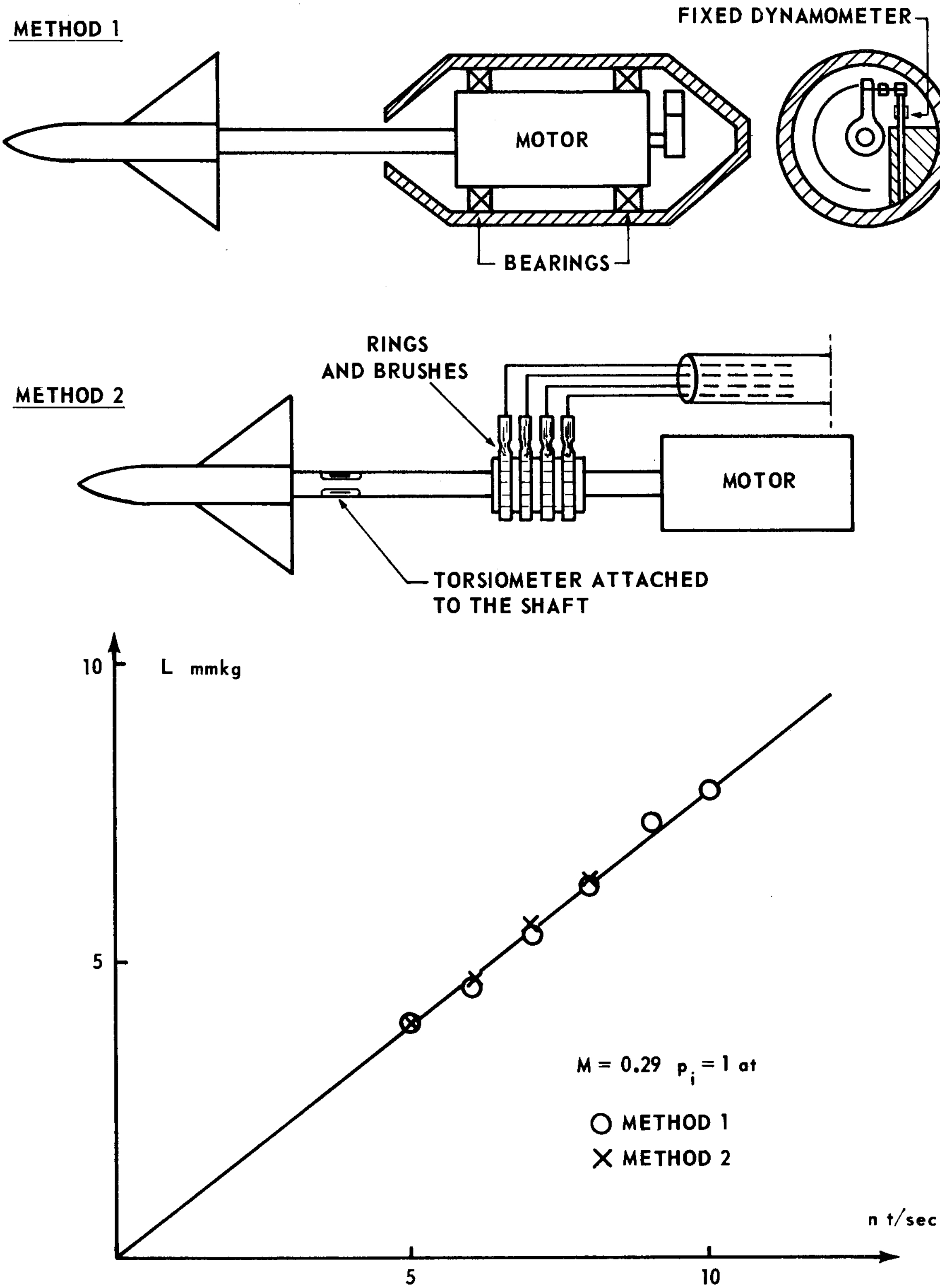
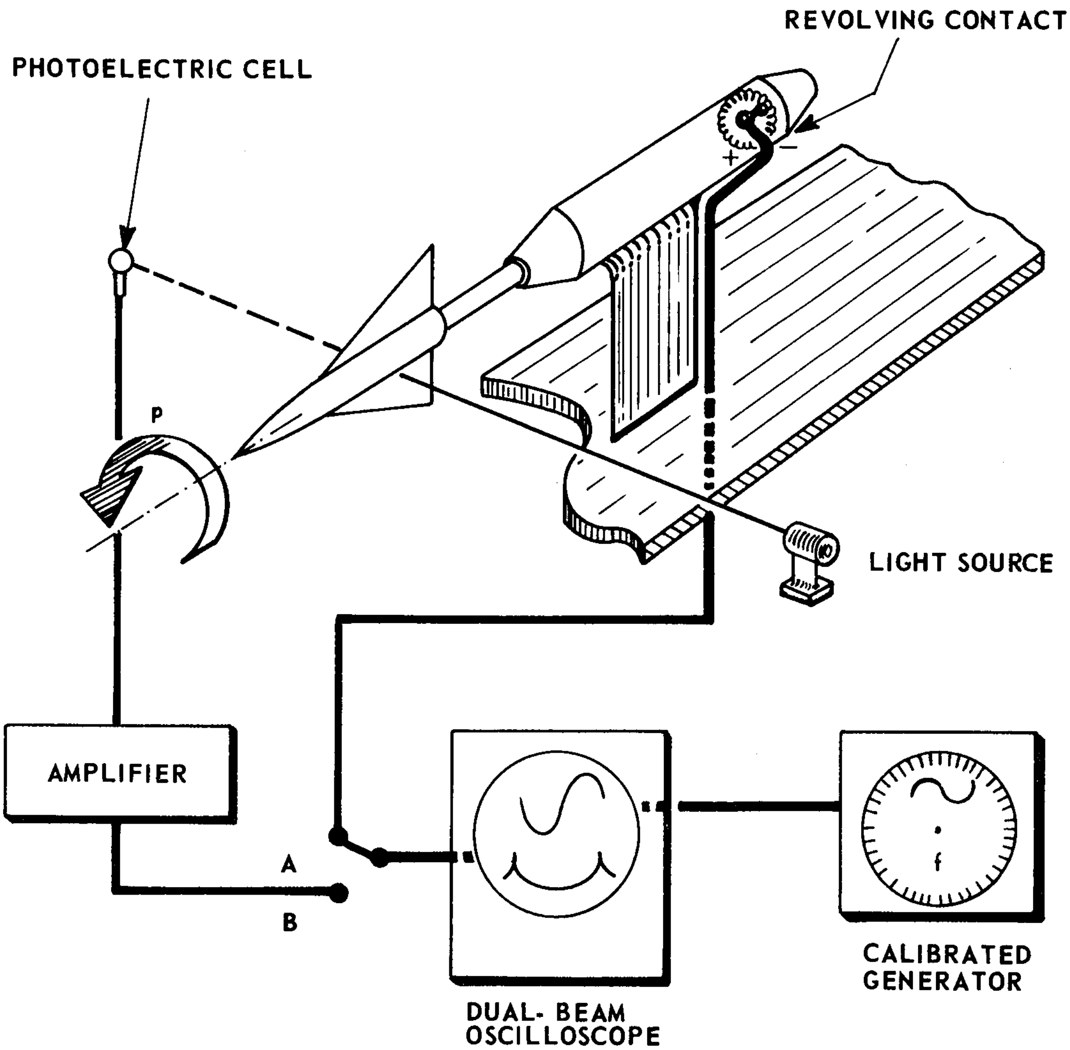
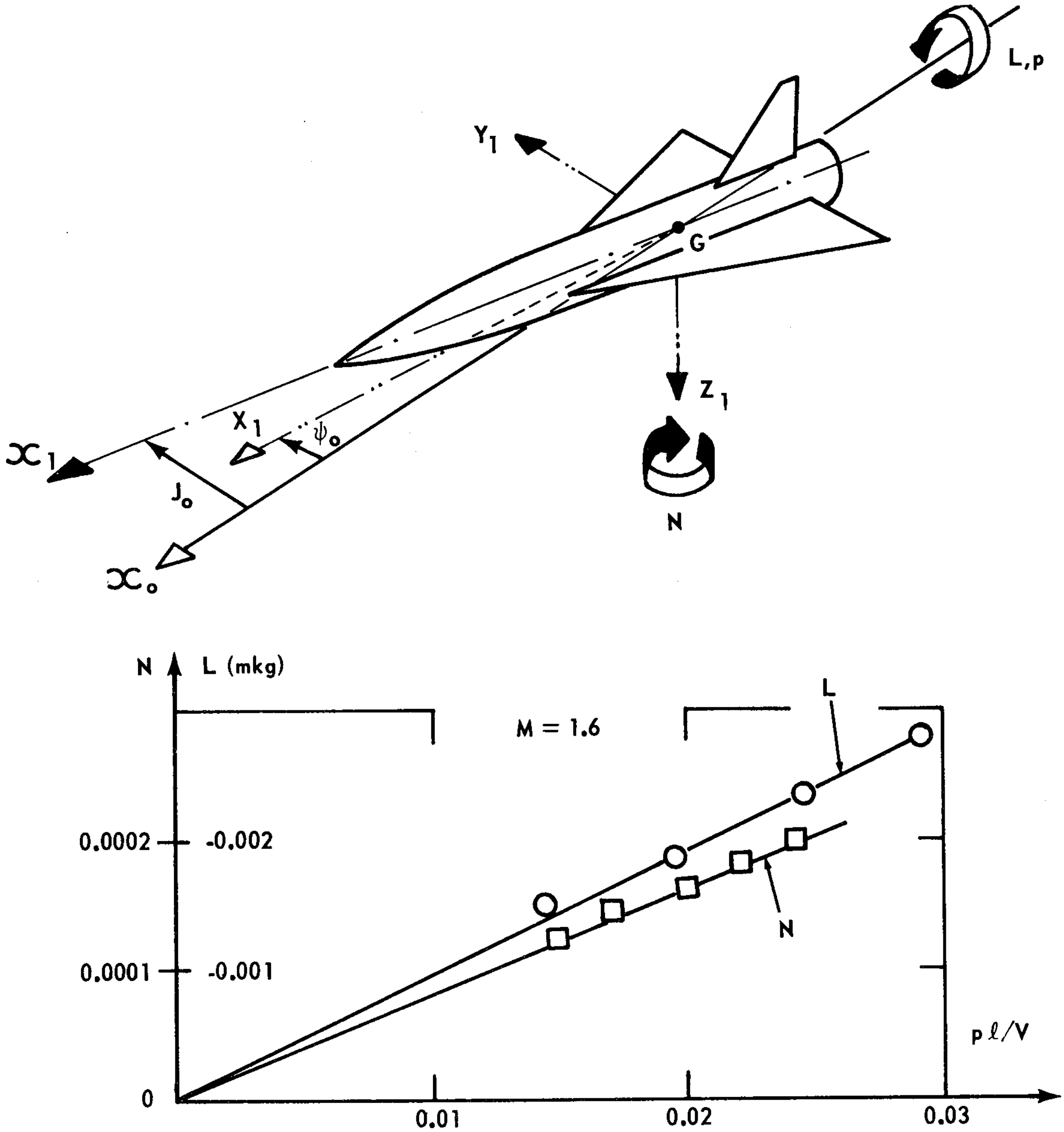


Fig. 1 Example of quantities measured at uniform rotation in roll



SOLUTION A : $p = 2\pi f$
SOLUTION B : $p = 2k\pi f$ (CASE REPRESENTED $k = 2$)

Fig. 2 Measurement and control of the rotation velocity



$$C_n = \frac{\Delta N^{(\pm)}}{pV^2 S\ell} \overbrace{- \frac{1}{2}\left(k_1 n^2 + C_{n_i}\right)\delta\psi^{(\pm)} - kC\ell}^{\text{CORRECTIONS}}$$

$$C_{\ell_p} = \frac{C\ell}{p\ell/v} \quad , \quad C_{n_p} = \frac{C_n}{p\ell/v}$$

Fig. 3 Measurement of the derivative C_{n_p}

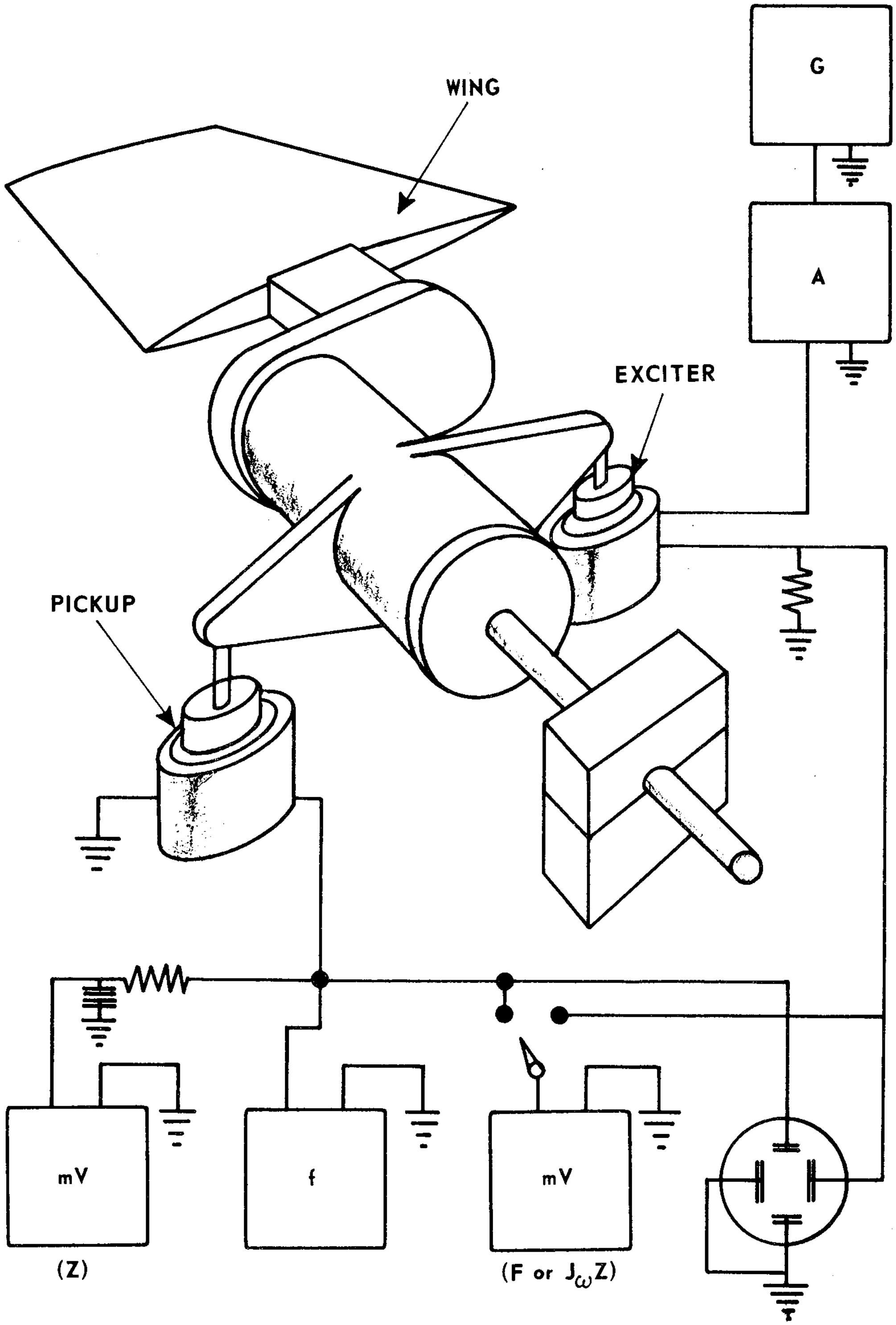


Fig. 4 Diagram of the free and continuous oscillation assembly taken from AGARD Report 220 by R. Dat

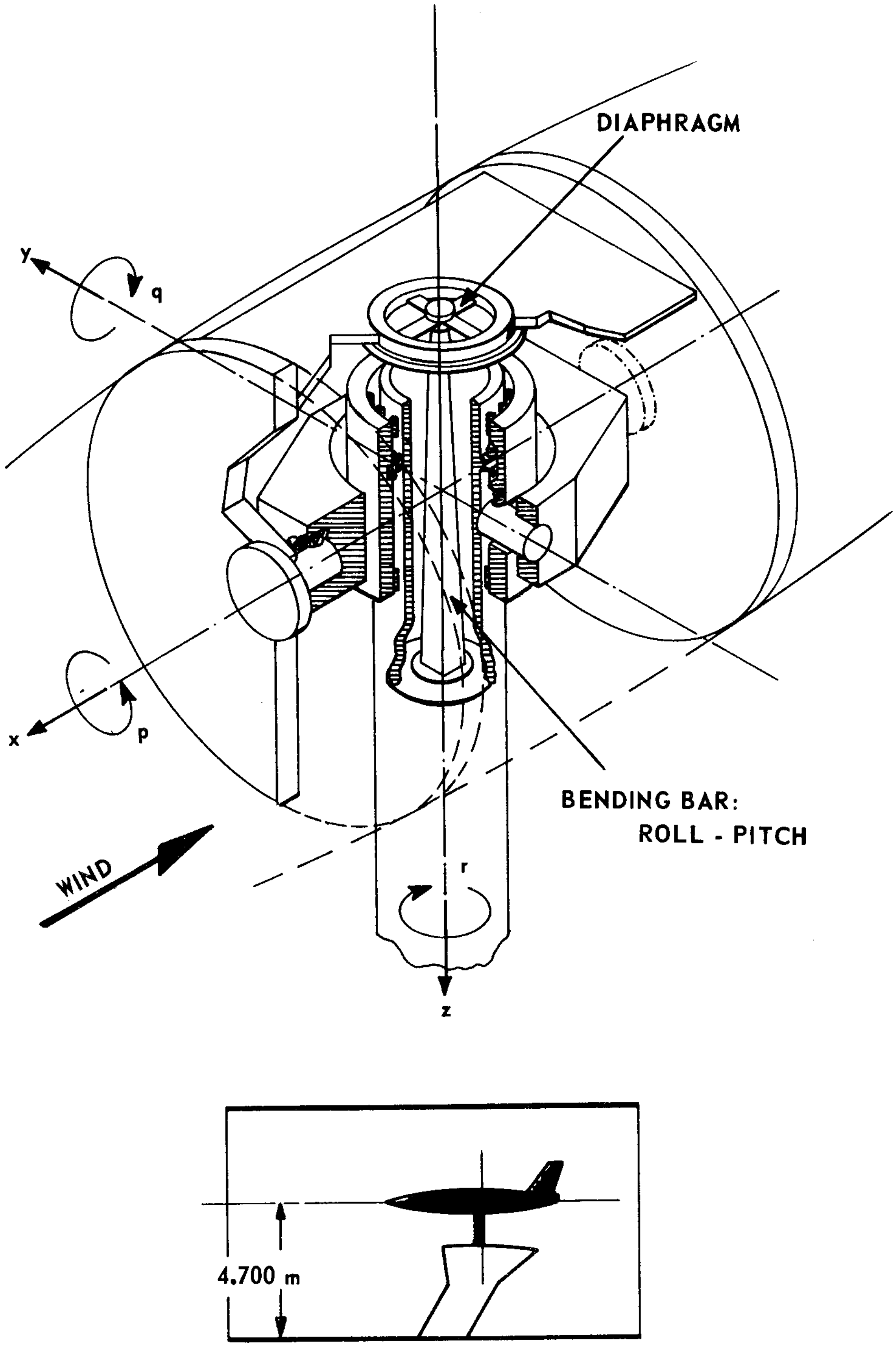


Fig. 5 Diagram of the free oscillation assembly of the great wind tunnel of the O. N. E. R. A. at Modane

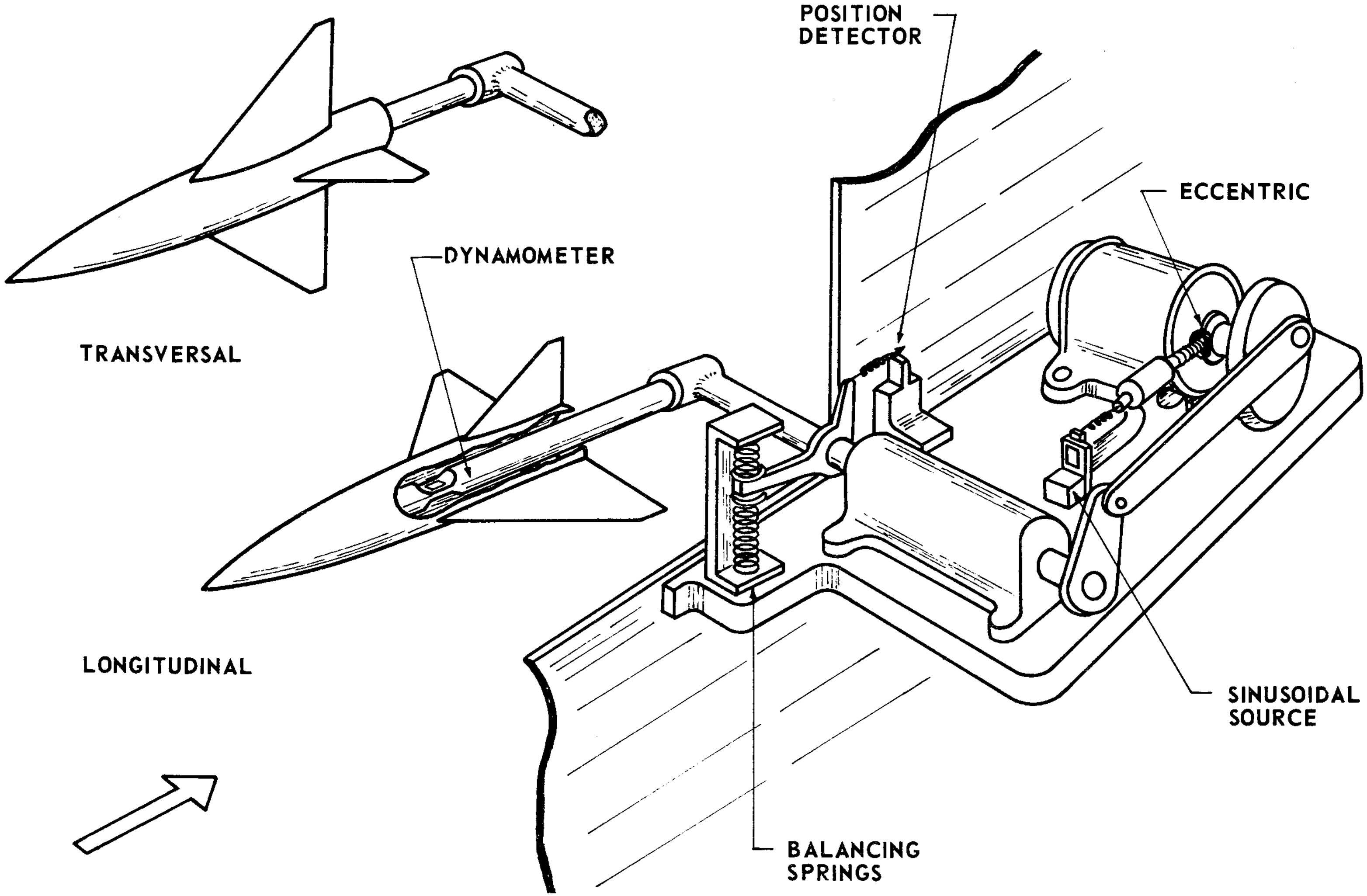
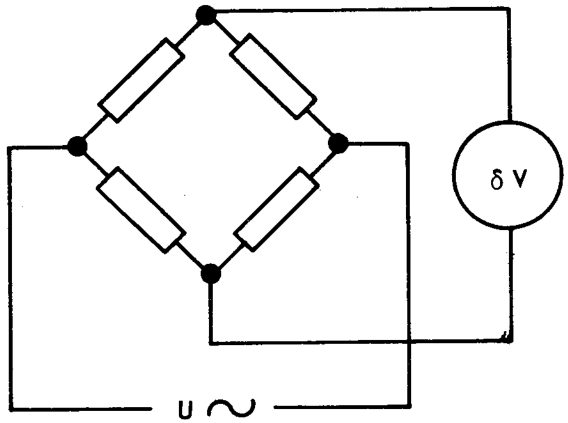


Fig. 6 Diagram of a pitch-yaw continuous oscillation balance

RESISTANCE VARIATION OF A GAUGE

$$\delta R_o \left[\sin(\omega t + \varphi) + \epsilon(n\omega t) \right]$$



CONTINUOUS GALVANOMETER

$$U_o \sin \omega t \longrightarrow \left\{ \begin{aligned} \delta V_{(o)} &= kU_o\, \delta R_o \left[\sin\omega t \sin(\omega t + \varphi) + \text{------} \right] \\ &= \underline{kU_o\, \delta R_o \left[\cos\varphi - \cos(2\omega t + \varphi) \right] + \ldots\ldots} \end{aligned} \right.$$

$$U_o \cos \omega t \longrightarrow \delta V_{\left(\frac{\pi}{2}\right)} = \underline{kU_o\, \delta R_o \left[\sin\varphi + \sin(2\omega t + \varphi) \right] + \ldots\ldots}$$

Fig. 7 Principle of measuring with strain gauges

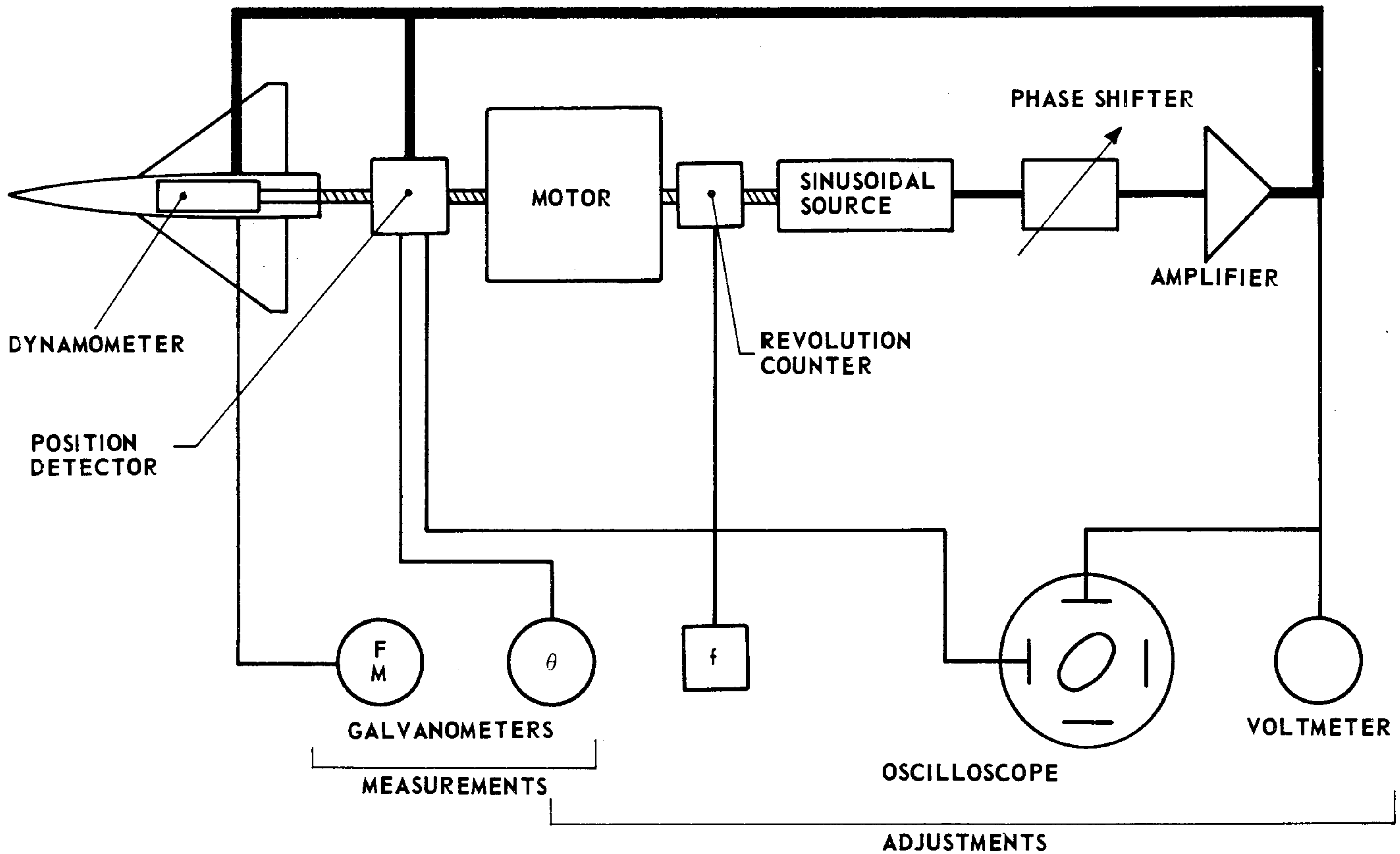


Fig. 8 Diagram of operation

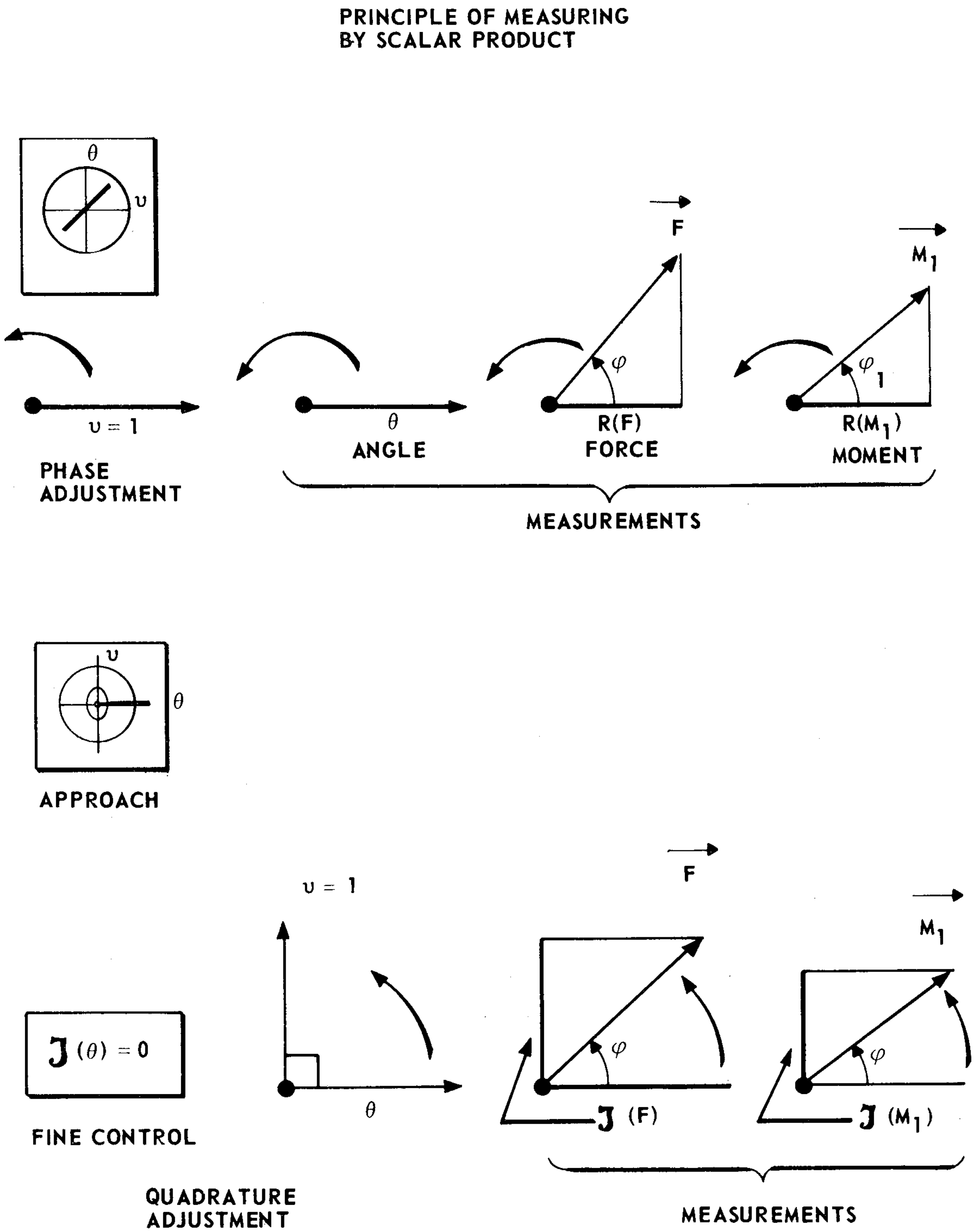


Fig. 9 Vectorial representation of motion and stresses

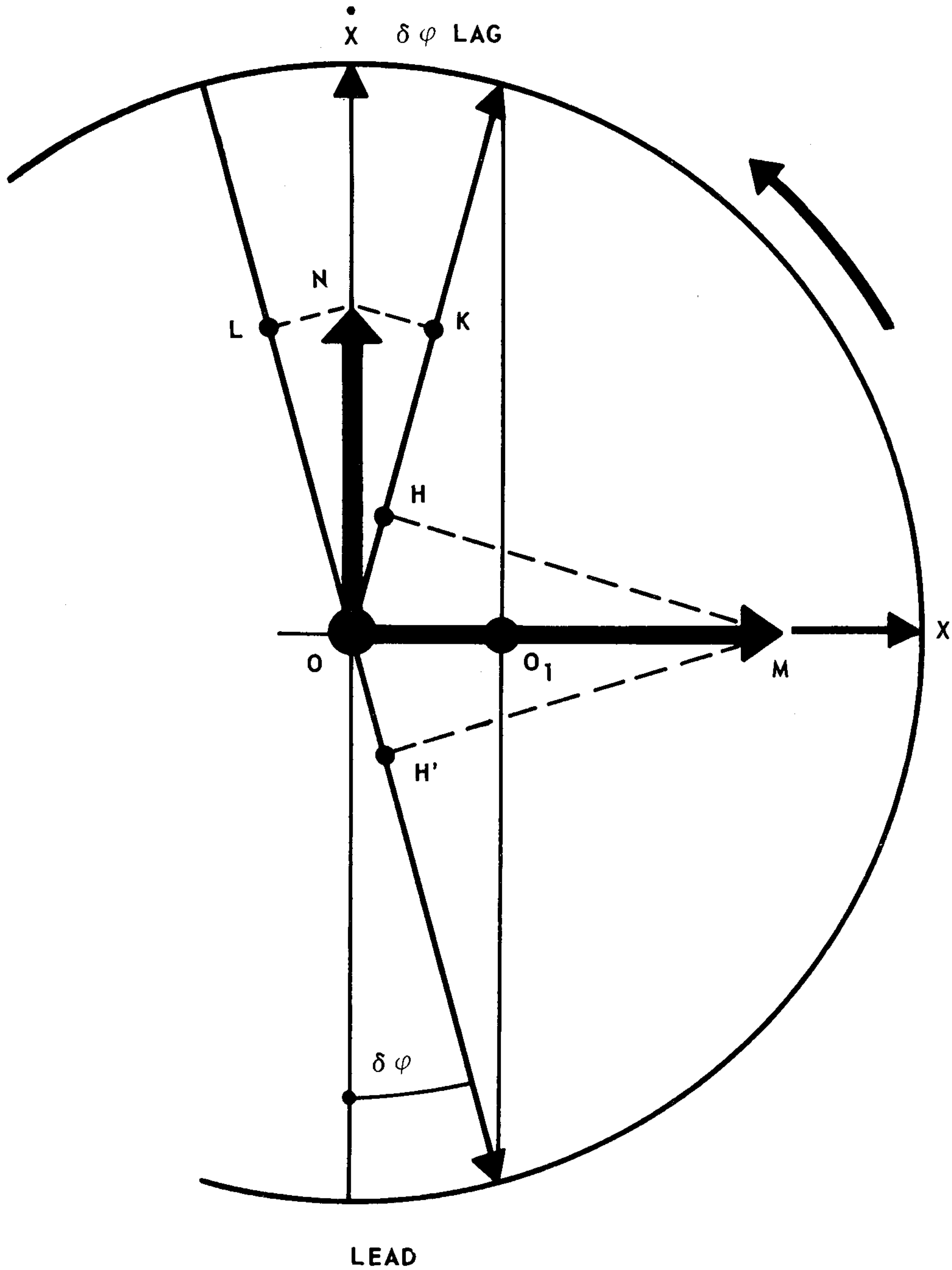


Fig. 10 Correction of the phase error in the quadrature adjustment

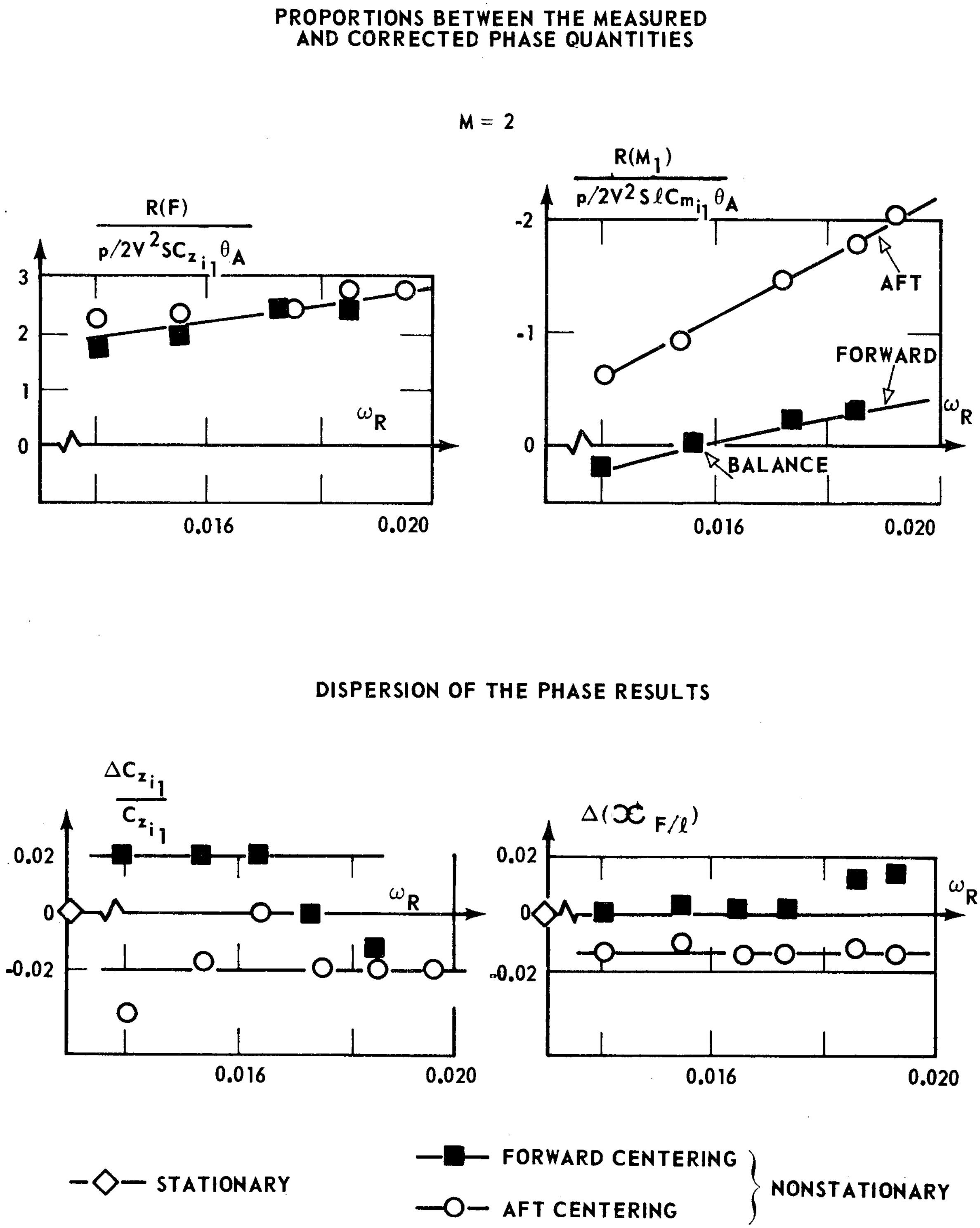
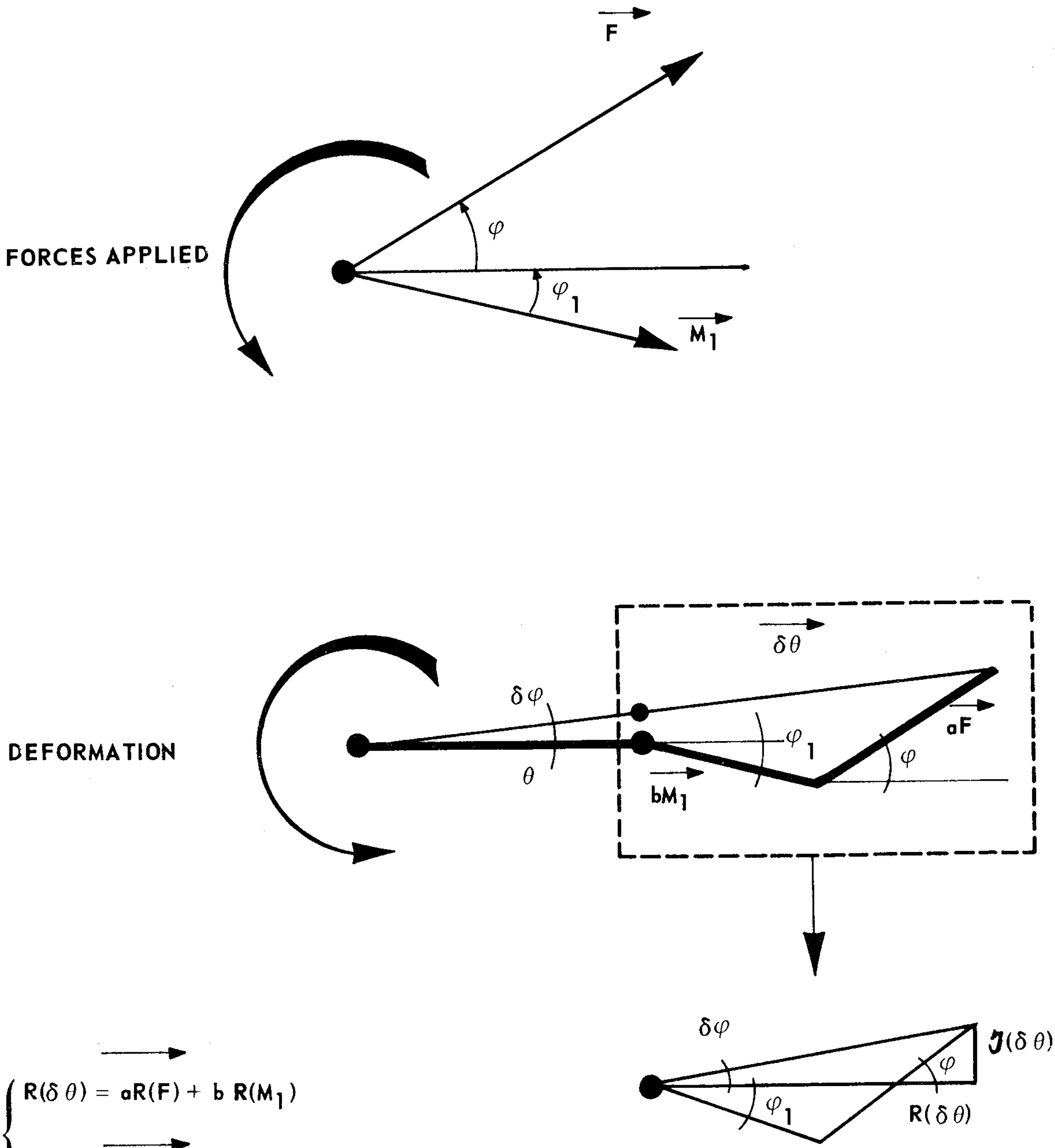


Fig. 12 Proportions and dispersion of the phase results

DEFORMATION CORRECTION OF THE STING



$$\begin{cases} \overrightarrow{R(\delta\theta) = aR(F) + b\,R(M_1)} \\ \overrightarrow{\mathfrak{J}(\delta\theta) = a\mathfrak{J}(F) + b\,\mathfrak{J}(M_1)} \end{cases}$$

Fig. 11 Vectorial representation of the deformation of the dynamometric sting

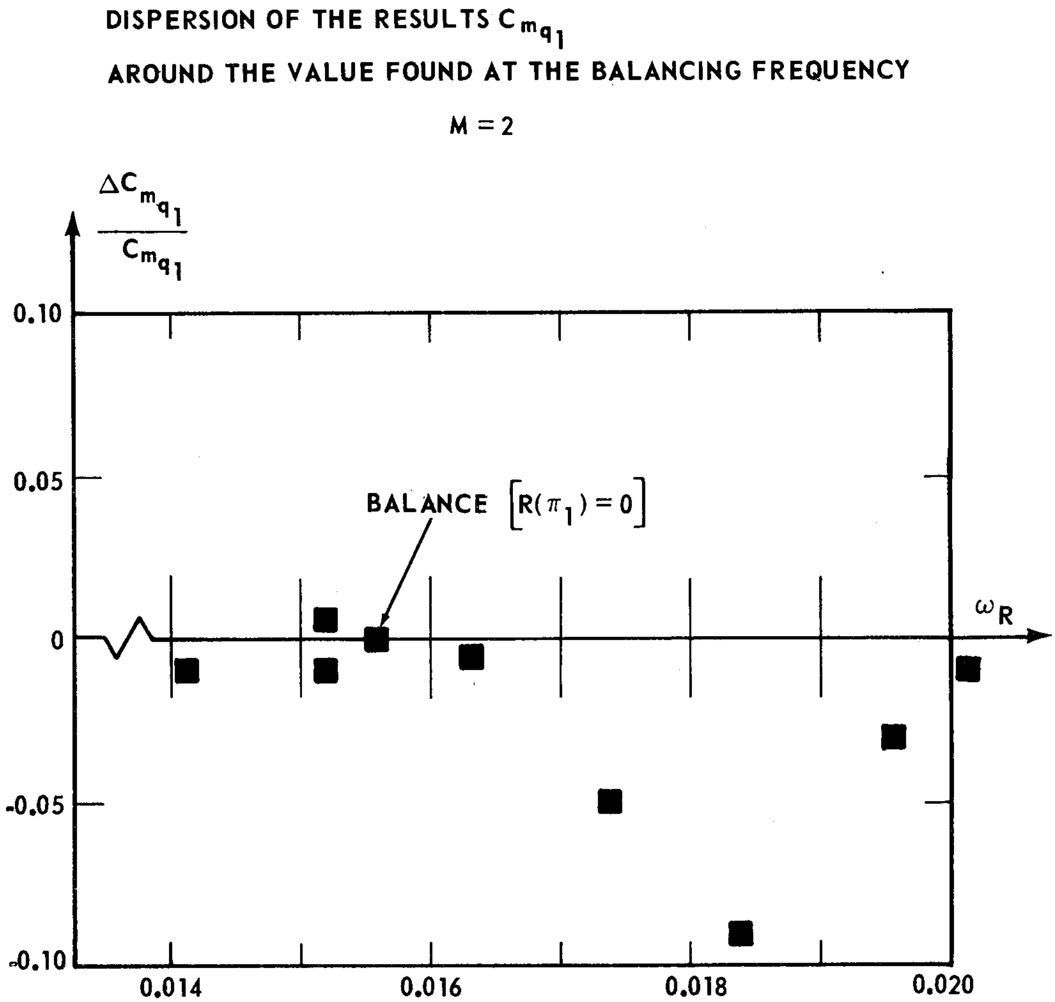
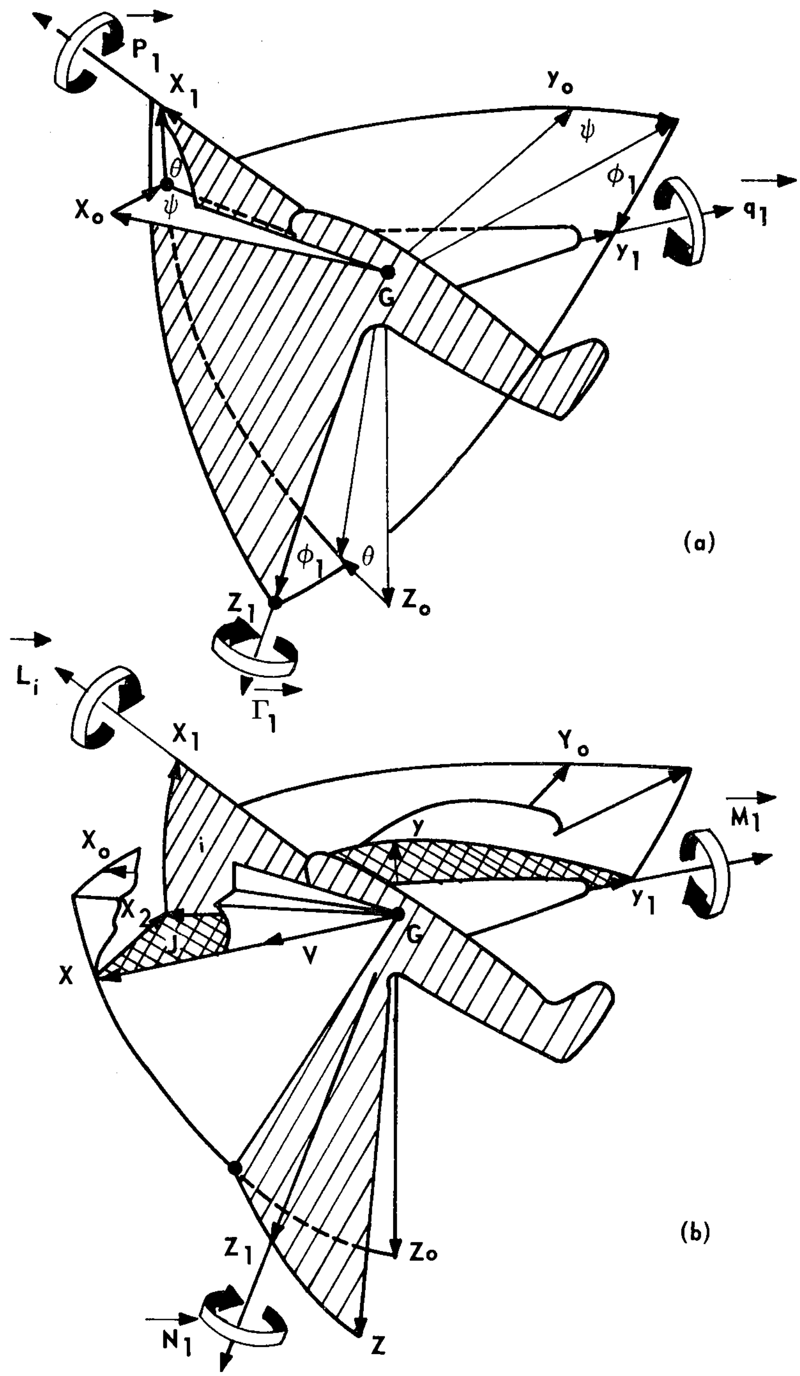


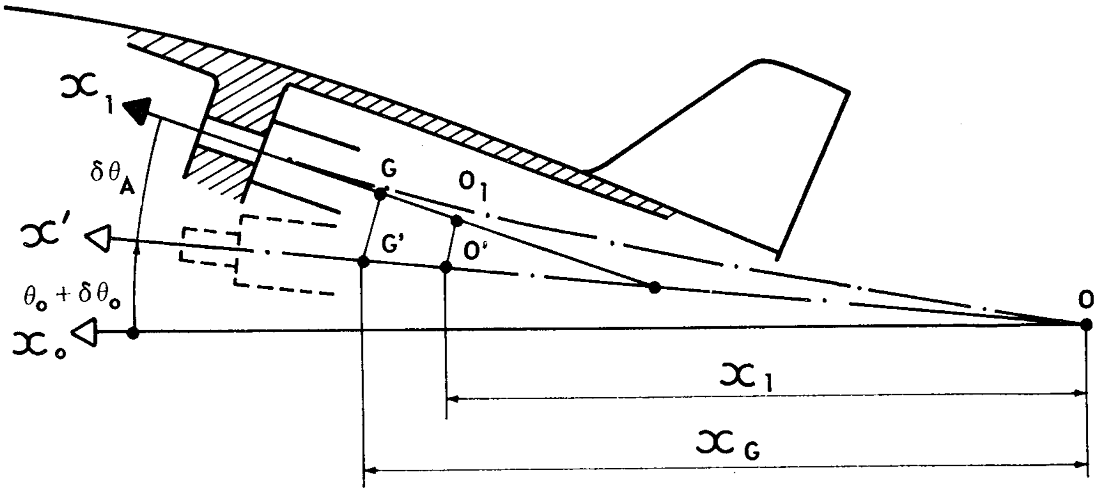
Fig. 13 Dispersion of the results $C_{m_{q_1}}$



REFERENCE TRIHEDRONS, NOTATIONS GALILEO TRIHEDRON $Gx_o y_o z_o$; Gz_o IS VERTICAL, AIRPLANE TRIHEDRON, $GX_1 Z_1$ IS THE SYMMETRICAL PLANE. THE ANGLES SHOWN ARE POSITIVE; THE CURVED ARROWS INDICATE THE POSITIVE DIRECTION OF THE COMPONENTS OF ANGULAR VELOCITY (a) AND AERODYNAMIC MOMENT (b).

Fig. 14 Reference trihedrons; notation

AMPLITUDE CORRECTIONS DUE TO DEFORMATION

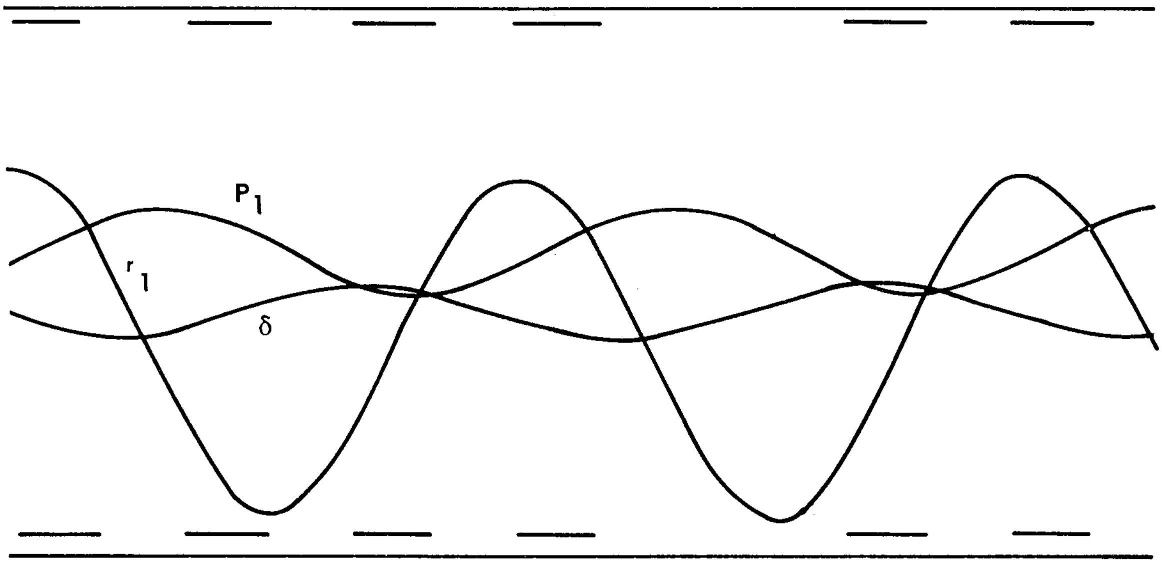


$\delta\theta_G = \dfrac{G'G}{OG'}$, $\delta\theta_{01} = \dfrac{O'O_1}{OO'}$ TRANSLATIONS

$\delta\theta_0$ (MEASURED) , $\delta\theta_A = \widehat{O\mathfrak{X}', O_1\mathfrak{X}_1}$ ROTATIONS

$$
\begin{bmatrix} \delta\theta_A \\ \delta\theta_G \\ \delta\theta_{01} \end{bmatrix}
=
\underbrace{\begin{bmatrix} \alpha_A & \beta_A & \Gamma_A \\ \alpha_G & \beta_G & \Gamma_G \\ \alpha_{01} & \beta_{01} & \Gamma_{01} \end{bmatrix}}_{\text{STATIC COEFFICIENTS}}
\begin{bmatrix} R(F) + J\mathfrak{J}(F) \\ R(M_1) + J\mathfrak{J}(M_1) \\ f^2 \end{bmatrix}
$$

Fig. 15 Diagram of sting deformation — Notation — Equations (Supplement)



$Z = 10{,}200$ ft

$V_c = 448$ kts

$$\boxed{\omega = 2.06}$$

$\frac{P}{\Delta} = 4.4$

$\varphi_{P_1, \delta}$ MEASURED $= -210$ deg

GYRO CORRECTION $= 7$ deg

$0.5 \text{ deg} \leqslant i \leqslant 1 \text{ deg}$

$\frac{R}{\Delta} = 1.91$

$\varphi_{r_1, \delta}$ MEASURED $= -112$ deg

GYRO CORRECTION $= 2.5$ deg

$5 \text{ deg} \leqslant \Phi_1 \leqslant 10 \text{ deg}$

Fig. 16 Example of readings taken in flight

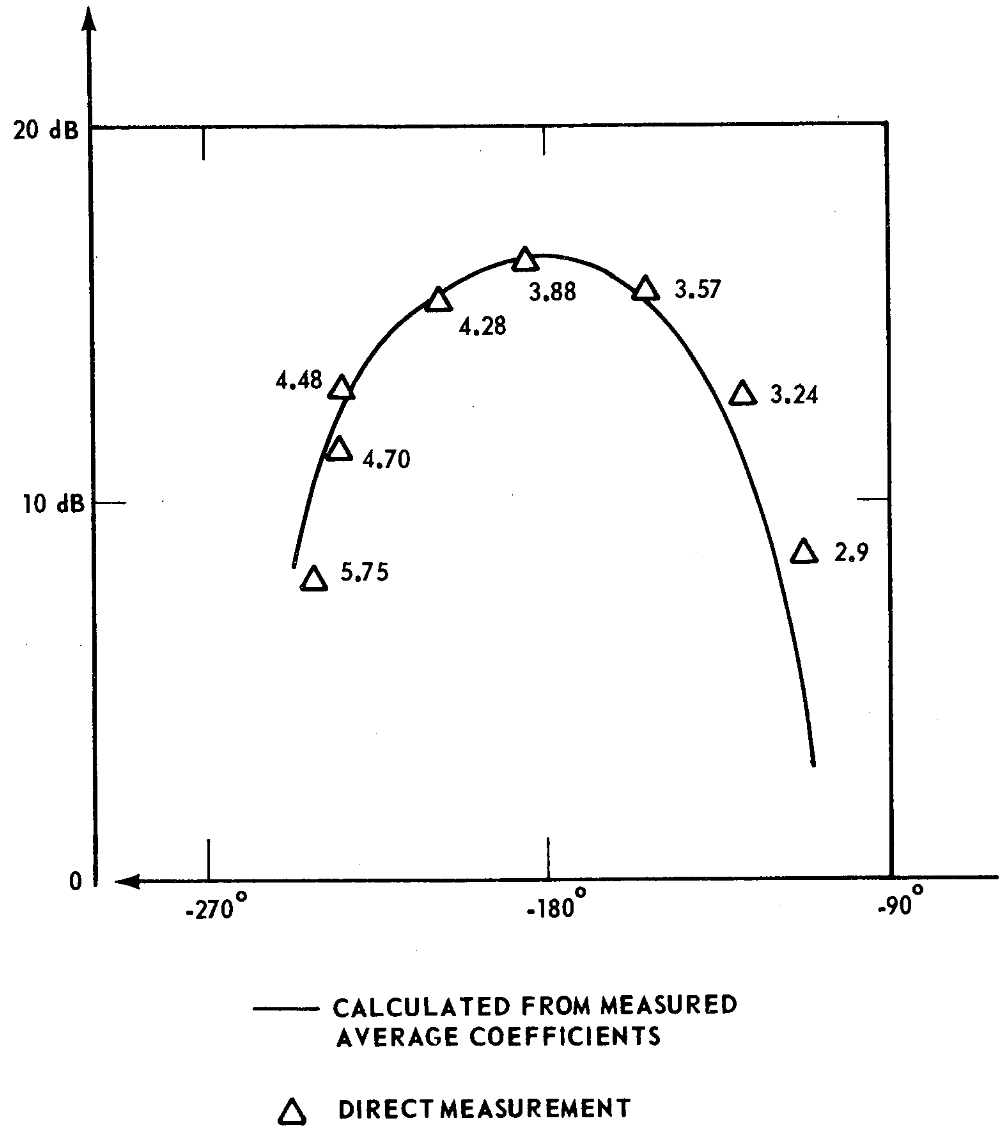


Fig. 17 Transfer function, r_1/δ

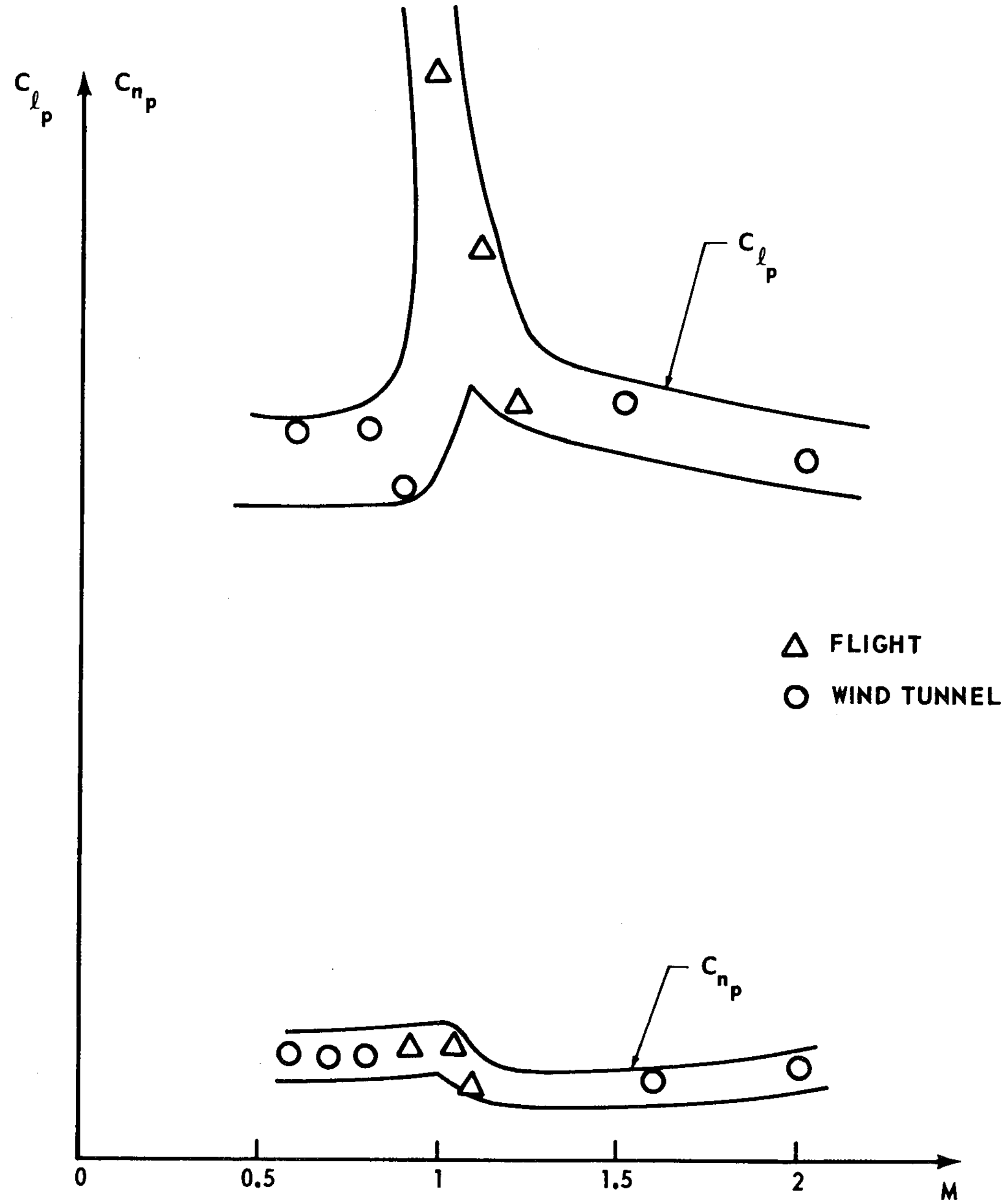


Fig. 18 Comparison of C_{ℓ_p}, C_{n_p}

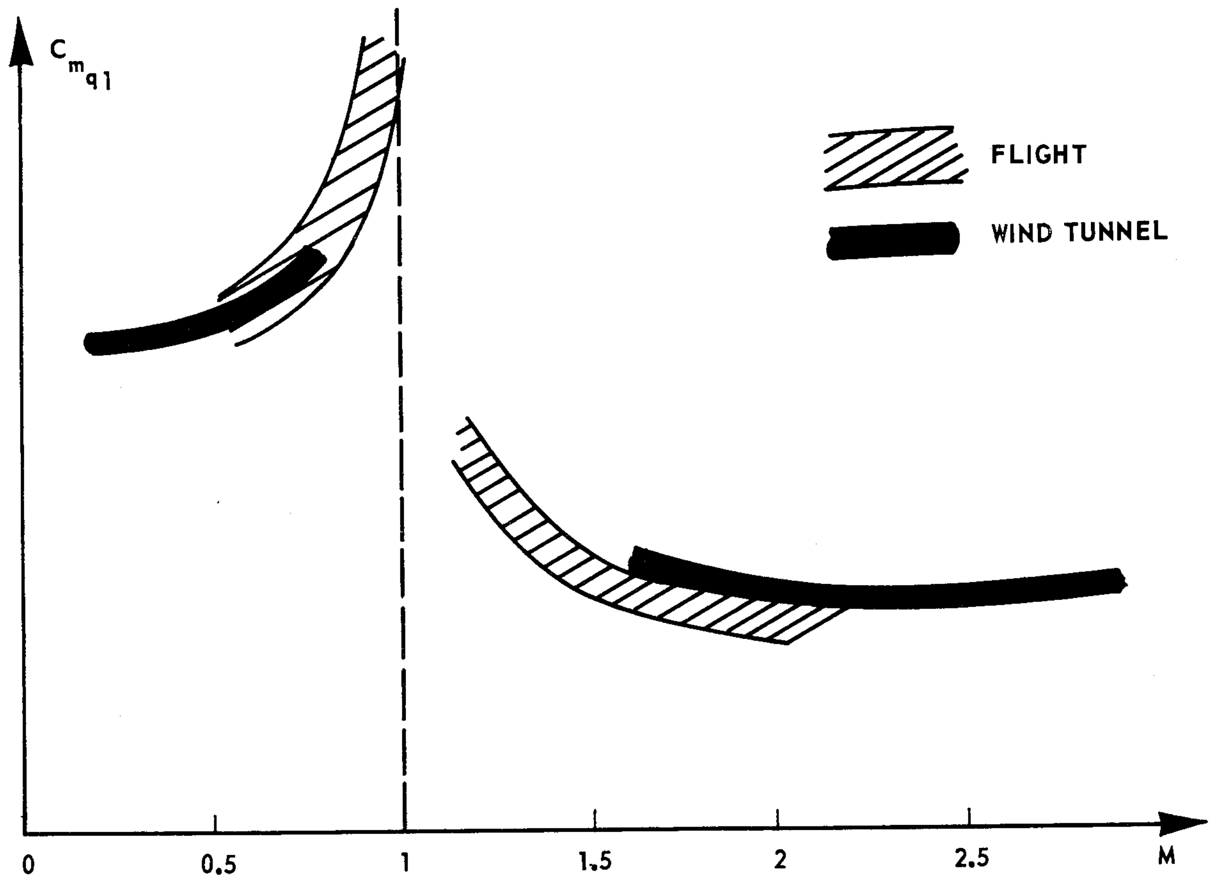


Fig. 19 Comparison of $C_{m_{q_1}}$ in flight and in the wind tunnel

SUPPLEMENT TO PART ONE

THEORY OF THE METHOD OF MEASURING IN WIND TUNNELS BY MEANS OF FORCED OSCILLATIONS

1. NOTATIONS

Specific mass of the air	ρ
Velocity of the flow	V
Mach number	M
Reference cord	ℓ
Reference surface	S
Frequency of the oscillations	f
Angular frequency ($2\pi f$)	ω
Frequency parameter ($\pi f \ell / V$)	ω_R
Angular velocity of pitch	q_1
Angular velocity of yaw	r_1
Angle of incidence	i
Angle of sideslip	j
Mass of the model including the weighed part of the dynamometer	m
Radius of gyration around Gy_1	r_B
Reference axes relative to the representative balance	$Ox_0y_0z_0$
Oscillation axis	Oy_0
Geometric axis of the sting without deformation	Ox_0
Abcissa of the center of inertia of the mass m	x_G

Abcissa of the center of reduction of the moments M_1 or N_1		x_1
Velocity of 0_1		w_1
Elongation of the oscillatory motion		Θ
Corresponding amplitude without deformation		θ_0
Amplitude of the angle described by the longitudinal axis of the model (aerodynamic incidence)		θ_A
Reduced amplitude of the motion of the center of gravity of the model $\left(z_G/x_G\right)$		θ_G
Reduced amplitude of the motion of the axis of measurement		θ_{01}

Amplitude Increments Due to Deformations of the Balance

(a) measured		$\delta\theta_0$
(b) not measured caused by measured stresses		
slope of the extremity of the sting		$\delta\theta_A$ or h_x
relative slope of G	$G'G/OG'$	$\delta\theta_G$ or h
relative slope of O	$O'O_1/OO'$	$\delta\theta_{01}$ or h_{01}

Quantities in phase with the imposed motion

Stress	R(F)
Moment	$R(M_1)$
Amplitude increment	$R(h)$, $R\left(h_x\right)$

Quantities in quadrature with the imposed motion

Stress	$J(F)$
Moment	$J(M_1)$
Amplitude increment, etc.	$J(h)$, $J\left(h_x\right)$
Symbol of imaginary numbers	$\bar{j}$
Successive derivatives with respect to time	$\dot{\theta}$, $\ddot{\theta}$

2. EQUATIONS OF MOTION OF THE MODEL

In the case of longitudinal measurements, the stresses to which the model is submitted and the values measured by the dynamometer are connected by the following relations:

$$\begin{bmatrix} C_{z_{i_1}} & C_{z_{q_1}} & C_{z_{\dot{i}_1}} & \dfrac{x_1}{V} & C_{z_{\dot{w}_1}} & \dfrac{x_1 \ell}{V^2} \\ & & & & & \\ C_{m_{i_1}} & C_{m_{q_1}} & C_{m_{\dot{i}_1}} & \dfrac{x_1}{V} & C_{m_{\dot{w}_1}} & \dfrac{x_1 \ell}{V^2} \end{bmatrix} \begin{bmatrix} \Theta_A \\ \dot{\Theta}_A \\ -\Theta_{01} \\ -\ddot{\Theta}_{01} \end{bmatrix}$$

$$= \begin{bmatrix} -\dfrac{F + m x_G \ddot{\theta} G}{\frac{1}{2}\rho V^2 S}\,, \\ \\ \dfrac{M_1 + m\left[x_G\left(x_G - x_1\right)\ddot{\theta}_G + r_B^2 \ddot{\theta}_A\right]}{\frac{1}{2}\rho V^2 S \ell} \end{bmatrix} \qquad (A.1)$$

The nature of the motion:

$$\Theta_A = \theta_A e^{\bar{j}\omega t}$$

leads to the relations:

$$\left.\begin{aligned} \dot{\theta}_A &= \bar{j}\omega\theta_A \\ \ddot{\theta}_A &= -\omega^2\theta_A \end{aligned}\right\} \tag{A.2}$$

and to two analogous relations for θ_{01} and θ_G. In consideration of (A.2) and by introducing the expression of the frequency parameter ω_R, equations (A.1) are transformed into:

$$\begin{bmatrix} \theta_A - 2\bar{j}\omega_R \frac{x_1}{\ell}\theta_{01} & 2\bar{j}\omega_R\theta_A & 4\omega_R\frac{x_1}{\ell}\theta_{01} \end{bmatrix} \begin{bmatrix} C_{z_{i_1}} & C_{m_{i_1}} \\ C_{z_{q_1}} & C_{m_{q_1}} \\ C_{z_{w_1}} & C_{m_{w_1}} \end{bmatrix}$$

$$= \begin{bmatrix} -\dfrac{F + m\omega^2 x_G\theta_G}{\frac{1}{2}\rho V^2 S}, \\ \dfrac{M_1 - m\omega^2\left[x_G\left(x_G - x_1\right)\theta_G + r_B^2\theta_A\right]}{\frac{1}{2}\rho V^2 S\ell} \end{bmatrix} \tag{A.3}$$

For low frequency parameters $\left(\omega_R \leq 0.05\right)$ the terms in ω^2 can be disregarded and the expressions of equations (A.3) simplified:

$$\begin{bmatrix} \theta_A - 2\bar{j}\omega_R \frac{x_1}{\ell}\theta_{01} & 2\bar{j}\omega_R\theta_A \end{bmatrix} \begin{bmatrix} C_{z_{i_1}} & C_{m_{i_1}} \\ C_{z_{q_1}} & C_{m_{q_1}} \end{bmatrix}$$

$$= \begin{bmatrix} -\dfrac{F + m\omega^2 x_G\theta_G}{\frac{1}{2}\rho V^2 S}, \\ \dfrac{M_1 - m\omega^2\left[x_G\left(x_G - x_1\right)\theta_G + r_B^2\theta_A\right]}{\frac{1}{2}\rho V^2 S\ell} \end{bmatrix} \tag{A.4}$$

in which F and M_1 are complex quantities, as well as the corrected amplitudes of motion θ_A, θ_G, and θ_{01}.

Let

$$\theta_A = R(\theta_A) + \bar{j}J(\delta\theta_A)$$

$$\theta_G = R(\theta_G) + \bar{j}J(\delta\theta_G)$$

$$\theta_{01} = R(\theta_{01}) + \bar{j}J(\delta\theta_{01})$$

By substituting these expressions into Equation (A.4) and separating the real and imaginary parts, there results, after a few simplifications:

1. Real part

$$\begin{bmatrix} C_{z_{i_1}} \\ \\ C_{m_{i_1}} \end{bmatrix} = \begin{bmatrix} -\dfrac{R(F) + m\omega^2 x_G(\theta_G)}{\frac{1}{2}\rho V^2 S R(\theta_A)} \\ \\ \dfrac{R(M_1) - m\omega^2\left[x_G(x_G - x_1)R(\theta_G) + r_B^2 R(\theta_A)\right]}{\frac{1}{2}\rho V^2 S\ell R(\theta_A)} \end{bmatrix} \tag{A.5}$$

2. Imaginary part

$$\begin{bmatrix} 1 & \dfrac{J(\delta\theta_A)}{2\omega_R R(\theta_A)} & -\dfrac{x_1}{\ell}\dfrac{R(\theta_{01})}{R(\theta_A)} \end{bmatrix} \begin{bmatrix} C_{z_{q_1}} & C_{m_{q_1}} \\ C_{z_{i_1}} & C_{m_{i_1}} \end{bmatrix}$$

$$= \begin{bmatrix} -\dfrac{J(F) + m\omega^2 x_G J(\delta\theta_G)}{\frac{1}{2}\rho V^2 S\ell \; 2\omega_R R(\theta_A)} \\ \\ \dfrac{J(M_1) - m\omega^2\left[x_G(x_G - x_1)J(\delta\theta_G) + r_B^2 j(\delta\theta_A)\right]}{\frac{1}{2}\rho V^2 S\ell \; 2\omega_R R(\theta_A)} \end{bmatrix} \tag{A.6}$$

DISCUSSION

B. Etkin (Canada): Mr. Mathé has described the frequency response technique which he used to obtain stability derivatives from flight data. In principle the same results can be obtained by the use of transient test techniques, in which the input is a control surface pulse rather than a steady sinusoidal oscillation. The transient technique has been used in the U.S.A., I believe with some success. It has the obvious advantage of requiring less testing time. Would the speaker care to comment on whether he considered the method and, if so, why he prefers to use the frequency-response method?

Reply by P. L. Mathé: We have tried making the harmonic analysis of certain transient responses of an airplane. To put it briefly, the results were very disappointing. There is absolutely no comparison in accuracy. You have seen the transfer function R/S which we had both calculated and measured; it represents a frequency band extending from half to twice the frequency of the airplane itself. This is quite a large frequency band. If a harmonic analysis of the response is applied to a triangular or staggered solicitation, there is a much greater dispersion of the points which does not allow the coefficients to be found. There are perhaps special cases in which these coefficients could be obtained; but in the transversal case in particular, we have never been able to obtain test results aligned well enough to determine the coefficients.

H.H.B.M. Thomas (U.K.): Our experience in the U.K. suggests that, whatever method of analysis or excitation is used, the limit of accuracy of results in terms of derivatives is set by the instruments. A considerable effort has been put into improvement of these, but my impression is that the return is not in proportion. Would Mr. Mathé care to comment?

Reply by P. L. Mathé: It is true that all of these problems go back to the question of measuring instruments, and I believe that there is much progress to be made in this domain. Moreover, there is progress to be made in the field of the measurements themselves; that is to say, a transition to measurements on magnetic tape, which would make subsequent interpretations easier.

ADDENDUM

AGARD SPECIALISTS' MEETING

on

STABILITY AND CONTROL

Complete List of Papers Presented

Following is a list of the titles and authors of the 41 papers presented at the Stability and Control Meeting held in Brussesl in April 1960, together with the AGARD Report number covering the publication of each paper.

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13. ABSTRACT

The first part of this report, which concerns wind tunnel research on aerodynamic derivatives, amplifies previous overall studies by examining critically the methods and facilities generally used at the present time for such work. Some progress is reported and some results thus made possible are presented. The second part is devoted to flight tests undertaken by a French firm after development of prototype apparatus.